A set-theoretical investigation of Pāṇini's *Śivasūtras*

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ABSTRACT. In Pāṇini's grammar one finds the *Śivasūtras*, a table which defines the natural classes of phonological segments in Sanskrit by intervals. We present a formal argument which shows that, using his representation method, Pāṇini has chosen an optimal way of ordering the phonological segments to represent the natural classes. The argument is based on a strict set-theoretical point of view depending only on the set of natural classes and does not explicitly take into account the phonological features of the segments, which are, however, implicitly given in the way a language clusters its phonological inventory. Moreover, the argument is so general that it allows one to decide for each set of sets whether it can be represented with Pāṇini's method. Actually, Pāṇini had to modify the set of natural classes in order to define it by the *Śivasūtras* (the segment *h* plays a special role). We show that this modification was necessary and, in fact, the best possible modification. We discuss how every set of classes can be modified in such a way that it can be defined in a *Śivasūtra*-style representation.[1]

0.1 Pāṇini's *Śivasūtras*

Pāṇini's grammar is recognized as a consistent theoretical analysis of spoken Sanskrit (*bhāṣā*) of the time of its origin (ca. 350 BC). The *Śivasūtras* form the first part of it (a short survey of the structure of Pāṇini's grammar can be found in Kiparsky (1994)) and define the phonological segments of the language and their grouping in natural phonological classes, called *pratyāhāras*. In the *Aṣṭādhyāyī*, a system of about 4000 grammatical rules or rule elements forming the central part of his grammar, Pāṇini refers to 42 of the *pratyāhāras* in hundreds of rules.

The *Śivasūtras* state 42 phonological segments and consist of 14 *sūtras* (rows in table 1), each of which consists of a sequence of phonological segments (transcribed with small letters) bounded by a marker (transcribed with a capital letter),

[1]This approach fits naturally in the framework of Formal Concept Analysis, since the investigated graphs are formal concept lattices. The proofs of the presented propositions can be found in my thesis, which will appear in 2003, and are sketched in an the extended version of the present paper.

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1.	a	i	u			Ṇ
2.				ṛ	ḷ	K
3.		e	o			Ṅ
4.		ai	au			C
5.	h	y	v	r		Ṭ
6.					l	Ṇ
7.	ñ	m	ṅ	ṇ	n	M
8.	jh	bh				Ñ
9.			gh	ḍh	dh	Ṣ
10.	j	b	g	ḍ	d	Ś
11.	kh	ph	ch	ṭh	th	
			c	ṭ	t	V
12.	k	p				Y
13.		ś	ṣ	s		R
14.	h					L

Table 1: Pāṇini's *Śivasūtras*

called *anubandha*. Phonological classes are denoted by abbreviations, called *pratyāhāras*, consisting of a phonological segment and an *anubandha*. The elements of such a class are defined by the *Śivasūtras* given in table 1 and are the continuous sequence of phonological segments starting with the given segment and ending with the last segment before the *anubandha*. Table 2 gives an example of a *pratyāhāra*.

1.	a	**i**	**u**			Ṇ
2.				**ṛ**	**ḷ**	K
3.		**e**	**o**			Ṅ
4.		**ai**	**au**			C
5.	h	y	v	r		Ṭ

Table 2: Example of a *pratyāhāra*: iC = {i,u,ṛ,ḷ,e,o,ai,au}

In this way 285 *pratyāhāras* can be constructed, which is more than the 42 actually needed by Pāṇini, but it is still a small number compared to the number of all classes that can be formed from the phonological segments, which is $2^{42} > 4 \cdot 10^{12}$.

As Kornai (1993) points out clearly, the task of characterizing a phonological system of a language is to specify the segmental inventory, phonological rules and the set of natural classes of phonological segments which allow generalized rules. The set of natural classes is externally given by the phonological patterning of a language and it always meets two conditions: it is small compared to the set of unnatural classes and the nonempty intersection of two natural classes is a natural class itself.

Kornai stresses that the representation device used for the notation of the classes must make it easier to use natural classes than unnatural ones (e.g. the complement of a natural class is generally unnatural). Contemporary phonological theories build up a structured system of phonological features which are used to characterize the natural classes. Instead of referring to phonological features in order to define a phonological class, Pāṇini refers to intervals in a linear order of the phonological segments. His method of defining the natural classes by *pratyāhāras* – intervals of the *Śivasūtras* – meets the required conditions.

The phonological classes of a grammar are mutually related: classes can be subclasses of other classes, two or more classes can have common elements, etc. These connections are naturally represented in a hierarchy. A *Śivasūtra*-style representation encodes such connections in a linear form.[2] An aim of this paper is to determine the conditions under which a set of sets does have a *Śivasūtra*-style linear representation.

The rest of the paper is organized as follows: In section 0.2 a general formalization of Pāṇini's *Śivasūtra*-style representation of phonological classes is given. Furthermore, the main questions which will be answered in the course of the paper are raised. Section 0.3 explains how the Hasse-diagrams of sets of subsets determine whether a *Śivasūtra*-style representation of natural classes exists. Since some results of graph theory are needed, a brief introduction to planar graphs is given. Finally, in section 0.4 a procedure is presented which constructs an optimal *Śivasūtra*-style representation of a set of natural classes if it exists. This section ends with the proof that Pāṇini has chosen a perfect *Śivasūtra*-style representation. The whole approach is based only on a set-theoretical investigation of the set of natural classes used in Pāṇini's grammar of Sanskrit. No external – phonological – arguments are involved.

0.2 General definitions and the main questions

Definition 0.2.1 *A* well-formed *Śivasūtra*-alphabet *(short* S-alphabet*) is a triple* $(\mathcal{A}, \Sigma, <)$ *consisting of a finite alphabet* $\mathcal{A}$ *and a finite set of markers* Σ *(such that* $\mathcal{A} \cap \Sigma = \emptyset$*), and a total order* $<$ *on* $\mathcal{A} \cup \Sigma$*.*

Definition 0.2.2 *A subset* T *of the alphabet* $\mathcal{A}$ *is* S-encodable in $(\mathcal{A}, \Sigma, <)$ *iff there*

[2] Since Pāṇini's grammar was designed for oral tradition, the linear form of the *Śivasūtras* was a prerequisite.

exists $a \in \mathcal{A}$ *and* $M \in \Sigma$*, such that* $T = \{b \in \mathcal{A} | a \leq b < M\}$. aM *is called the* pratyāhāra *or* S-encoding *of* T *in* $(\mathcal{A}, \Sigma, <)$.

Definition 0.2.3 *An S-alphabet* $(\mathcal{A}', \Sigma, <)$ corresponds *to a system of sets* $(\mathcal{A}, \Phi)$ *(where* Φ *is a set of subsets of* $\mathcal{A}$*) iff* $\mathcal{A} = \mathcal{A}'$ *and each element of* Φ *is S-encodable in* $(\mathcal{A}', \Sigma, <)$. *An S-alphabet which corresponds to* $(\mathcal{A}, \Phi)$ *is called an* S-alphabet of $(\mathcal{A}, \Phi)$. *A system of sets for which a corresponding S-alphabet exists is said to be* S-encodable.

For example, take the set of subsets

$$\Phi = \{\{d, e\}, \{b, c, d, f, g, h, i\}, \{a, b\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\} \quad (1)$$

of the alphabet $\mathcal{A} = \{a, b, c, d, e, f, g, h, i\}$: it is S-encodable and

$$a \; b \; M_1 \; c \; g \; h \; M_2 \; f \; i \; M_3 \; d \; M_4 \; e \; M_5 \quad (2)$$

is one of the corresponding S-alphabets. The *pratyāhāras* of Φ are: dM_5, bM_4, aM_1, fM_3, cM_5 and gM_2.

Definition 0.2.4 *An S-alphabet* $(\mathcal{A}, \Sigma, <)$ *of* $(\mathcal{A}, \Phi)$ *is said to be* optimal *iff there exists no other S-alphabet* $(\mathcal{A}, \Sigma', <')$ *of* $(\mathcal{A}, \Phi)$ *such that the set of markers* Σ' *has fewer elements than* Σ.

Looking at Pāṇini's *Śivasūtras* it is striking that the phonological segment *h* occurs twice, namely in *sūtra* 5 and *sūtra* 14. To model this phenomenon we will introduce the concept of *enlarging* an alphabet by duplicating some of its elements.[3]

$\hat{\mathcal{A}}$ is said to be an *enlarged alphabet* of $\mathcal{A}$ if there exists a surjective map $\vartheta : \hat{\mathcal{A}} \to \mathcal{A}$. It is clear that for every system of sets $(\mathcal{A}, \Phi)$ we can find an enlarged alphabet $\hat{\mathcal{A}}$ and a set of subsets $\hat{\Phi}$ with $\Phi = \{\vartheta(\varphi') : \varphi' \in \hat{\Phi}\}$ such that $(\hat{\mathcal{A}}, \hat{\Phi})$ is S-encodable. To achieve such an S-encodable system of sets $(\hat{\mathcal{A}}, \hat{\Phi})$ we enlarge $\mathcal{A}$ so that the sets of $\hat{\Phi}$ are disjoint. Then we arrange the sets of $\hat{\Phi}$ in a sequence and separate them by markers. The induced S-alphabet $(\hat{\mathcal{A}}, \hat{\Sigma}, \hat{<})$ then obviously corresponds to $(\hat{\mathcal{A}}, \hat{\Phi})$.

An S-alphabet of $(\hat{\mathcal{A}}, \hat{\Phi})$ will sometimes be called an *enlarged S-alphabet of* $(\mathcal{A}, \Phi)$. Since we always find a finite, enlarged S-alphabet of $(\mathcal{A}, \Phi)$, a minimally enlarged S-alphabet exists.

[3] Duplicating an element a means adding a new element a' to $\mathcal{A}$ and changing some of the occurrences of a in Φ to a'.

Definition 0.2.5 *An enlarged S-alphabet* $(\hat{\mathcal{A}}, \hat{\Sigma}, \hat{<})$ *of* $(\mathcal{A}, \Phi)$ *is said to be* perfect *iff it fulfills the following conditions: First, there exists no other enlarged S-alphabet* $(\tilde{\mathcal{A}}, \tilde{\Sigma}, \tilde{<})$ *of* $(\mathcal{A}, \Phi)$*, the alphabet* $\tilde{\mathcal{A}}$ *of which has fewer elements than* $\hat{\mathcal{A}}$ *and furthermore, as a secondary condition,* $(\hat{\mathcal{A}}, \hat{\Sigma}, \hat{<})$ *is optimal.*[4]

After this introduction of basic concepts the following questions will be investigated in the present paper:

1. Given a system of sets, is it possible to decide whether it is S-encodable?
2. If a system of sets is S-encodable, how can we construct an optimal corresponding S-alphabet?

And with respect to the special case of the phonological classes defined by Pāṇini's *Śivasūtras*:

3. Is the duplication of *h* in Pāṇini's *Śivasūtras* necessary?
4. Are Pāṇini's *Śivasūtras* perfect?

Kiparsky (1991) discusses questions 3 and 4 and affirms both, as I will do in what follows, using a different approach.

0.3 The existence of *Śivasūtra*-style representations of systems of sets

0.3.1 A Brief introduction to the theory of planar graphs

Throughout this paper we will need some basic knowledge about planar graphs, which will be briefly introduced in this section.[5] A *graph* G is a pair (V, E) consisting of a set of *vertices* V and a set of *edges* $E \subseteq V \times V$. *Paths* in graphs are defined in the natural way and *circles* are closed paths, as usual. *Directed graphs* are graphs the edges of which are directed.

A graph is a *plane graph* if its vertices are points in the Euclidean plane $\mathbb{R} \times \mathbb{R}$ and its edges are polygonal arcs in $\mathbb{R} \times \mathbb{R}$, such neither a vertex nor a point of an edge lies in the inner part of another edge. The Euclidean plane $\mathbb{R} \times \mathbb{R}$ is subdivided by a plane graph into *faces* (areas). Exactly one of this faces, the *infinite face*, is of unlimited size.

[4] Note that for every system of sets a perfect S-alphabet exists.

[5] The full details can be found in every introductory book on graph theory. In writing this paper Diestel (1997) proved to be especially helpful.

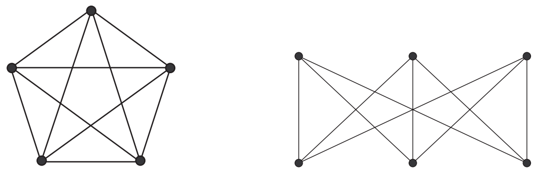

Figure 1: The graphs K^5 and $K_{3,3}$

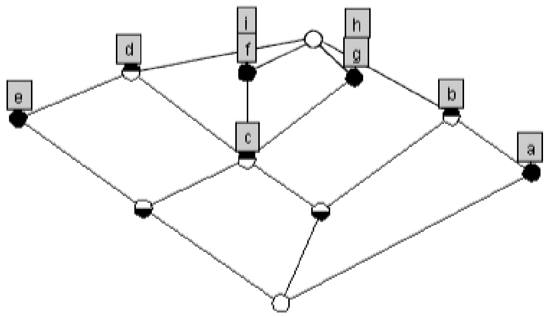

Figure 2: Hasse diagram of $(\mathcal{H}(\Phi), \supseteq)$, $\Phi = \{\{d,e\}, \{b,c,d,f,g,h,i\}, \{a,b\}, \{f,i\}, \{c,d,e,f,g,h,i\}, \{g,h\}\}$

If a graph is isomorphic to a plane graph, it is said to be *planar*. One of the most important criteria for the planarity of graphs is the criterion of Kuratowski, which is based on the notion of minors of a graph. A graph M is said to be a *minor* of a graph G if it can be arrived from G by first removing a number of vertices and edges from G and then contracting some of the remaining edges.

Proposition 0.3.1 (Criterion of Kuratowski) *A graph G is planar iff G contains neither a K^5 nor a $K_{3,3}$ as a minor (see figure 1).*

0.3.2 Plane Hasse-diagrams and S-encodability

Let $(\mathcal{A}, \Phi)$ be a system of sets as above, and let $\mathcal{H}(\Phi)$ be the set of all intersections of elements of $\Phi \cup \{\mathcal{A}\}$. $\mathcal{H}(\Phi)$ is partially ordered by the superset relation. A Hasse-diagram of a partially ordered set is a drawing of a directed graph whose vertices are the elements of the set and whose edges correspond to the upper neighbor relations determined by the partial order. The drawing must meet the following condition: if an element of the partially ordered set is an upper neighbor of another element, then its vertex lies above the vertex of the other one. In this paper we will stipulate that all edges are directed upwards.

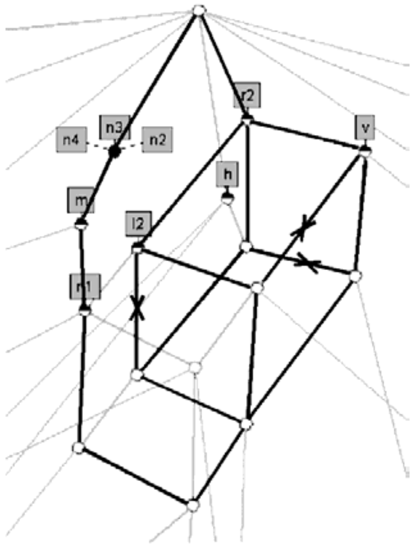

Figure 3: A section of the Hasse-diagram of the *pratyāhāras* used in the *Aṣṭādhyāyī* which has K^5 as a minor. The figure shows that the class memberships of the phonological segments *h*,*v* and *l* (denoted by *l2*) are independent of each other.

Figure 2 shows a drawing of the Hasse-diagram $(\mathcal{H}(\Phi), \supseteq)$ of our example system of sets $(\mathcal{A}, \Phi)$ from (1). All Hasse-diagrams in this paper are labeled economically as follows: for every $b \in \mathcal{A}$ the smallest set of $\mathcal{H}(\Phi)$ containing b (its b-set) is labeled. The set corresponding to a vertex of the diagram can be reconstructed by collecting all elements which are labeled to the vertex itself or to a vertex above. For example, in figure 2 the vertex labeled with c corresponds to the set $\{c, d, f, g, h, i\}$.

The Hasse-diagram of $(\mathcal{H}(\Phi), \supseteq)$ gives a first hint of the question whether $(\mathcal{A}, \Phi)$ is S-encodable:

Proposition 0.3.2 *If* $(\mathcal{A}, \Phi)$ *is S-encodable, then the Hasse-diagram of* $(\mathcal{H}(\Phi), \supseteq)$ *is a planar graph.*[6]

It follows as a corollary that a system of sets is not S-encodable whenever the Hasse-diagram of the corresponding set of intersections is not planar.

Together with Kuratowski's criterion 0.3.1 this answers question 3, since figure 3 shows a section of the Hasse-diagram of the *pratyāhāras* and their intersections,

[6]Due to the limited space, no proofs are given in this paper, but the results are illustrated by a number of examples.

which has K^5 as a minor.[7] Hence, Pāṇini was forced to duplicate at least one of the phonological segments. But it remains to prove that *h* is the best candidate for the duplication; this discussion will be postponed.

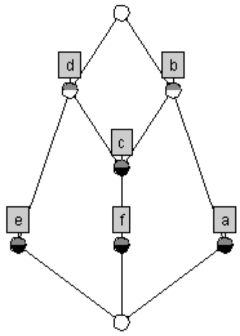

Figure 4: Hasse-diagram of the intersections of the sets $\{\{d,e\},\{a,b\},\{b,c,d\},\{b,c,d,f\},\{a,b,c,d,e,f\}\}$. It is plane, but there exists no corresponding S-alphabet.

The condition for S-encodable systems of sets given in proposition 0.3.2 is necessary but not sufficient, however. Figure 4 shows an example of a system of sets which is not S-encodable, although the Hasse-diagram of its intersection sets is planar. We need a second proposition to fully identify those systems of sets which are S-encodable.

If $(\mathcal{A},\Phi)$ is a system of sets which is S-encodable, then the boundary of the infinite face of a plane Hasse-diagram of $(\mathcal{H}(\Phi)\setminus\emptyset,\supseteq)$ is called the *S-graph of* $(\mathcal{A},\mathit{\Phi})$. It can be shown that the S-graph of $(\mathcal{A},\Phi)$ is fixed up to isomorphism, and is therefore independent of the chosen embedding of the Hasse-diagram in $\mathbb{R}^2$.

Proposition 0.3.3 *If $(\mathcal{A},\Phi)$ is S-encodable, then the Hasse-diagram of $(\mathcal{H}(\Phi),\supseteq)$ is a planar graph and the S-graph of $(\mathcal{A},\Phi)$ meets the following condition: For every $b\in\mathcal{A}$ the smallest set of $\mathcal{H}(\Phi)$ containing b (its b-set) is a vertex of the S-graph.*

In the example of figure 4 the f-set violates the condition of proposition 0.3.3 since in a plane Hasse-diagram of $(\mathcal{H}(\Phi)\setminus\emptyset,\supseteq)$ it would only touch inner faces. It is obvious that there exists no alternative embedding of the Hasse-diagram into $\mathbb{R}^2$ which would fulfill both conditions.

[7]The emphasized lines in the figure mark a way to arrive at the minor K^5: remove all edges which are not emphasized and contract those edges which are marked by arrows.

0.4 The construction of *Śivasūtra*-style representations of systems of sets

0.4.1 The boundary graph determines the S-alphabet

If $(\mathcal{A}, \Phi)$ is a system of sets which is S-encodable, then an S-alphabet $(\mathcal{A}, \Sigma, <)$ of $(\mathcal{A}, \Phi)$ can be found as follows: Take the labeled S-graph of $(\mathcal{A}, \Phi)$ and a path in it, that starts and ends at the vertex corresponding to $\mathcal{A}$. The path must meet the following conditions: First, for every $a \in \mathcal{A}$ the path passes the a-set at least once; second, none of the edges occurring more than once in the path is part of a circle in the S-graph. By looking at the S-graph as a subgraph of the directed Hasse-diagram, the edges of the path can be directed.

The S-alphabet, seen as a sequence of markers and elements of $\mathcal{A}$, can be constructed from the empty sequence by traversing the path from the beginning to the end: If a vertex is reached which is labeled with an a-set, then add a to the sequence, together with all other labels of the same vertex. If an edge is passed whose direction contradicts the traversal direction, a new, previously unused, marker element is added to the sequence, unless the last added element is already a marker. Finally, after the end of the path is reached, revise the sequence as follows: If an element of $\mathcal{A}$ appears more than once in the sequence, delete all occurrences of it except one, and if two markers happen to occur next to each other, remove one.

Applied to our small example (1) and the plane graph of its Hasse-diagram given in figure 2, we may choose the path illustrated in figure 5, which fulfills the required conditions. Traversing the path, we pass first the a-set and the b-set without using an edge against its destined direction. Now we move downwards and violate the direction of the edge, and therefore we have to add a marker to our sequence, so that it starts with $a\ b\ M_1$. Now moving upwards we collect the c and the g, but since the g- and the h-sets are identical we also have to collect the h. After this we move downwards again, and that is why we add a new marker. We again reach the c-set and add c a second time to our sequence. So far our sequence is $a\ b\ M_1\ c\ g\ h\ M_2\ c$, and if we continue we end up with the S-alphabet depicted in (2).

Note that this procedure does not yield a unique S-alphabet since we have several decisions to make: (a) If a vertex is labeled with more than one element, their order in the S-alphabet is arbitrary; (b) the ultimately deleted elements are arbitrarily chosen; (c) from the vertex labeled c we can either go to the vertex labeled gh or if; (d) the path can be traversed clockwise or anti-clockwise.

It can be shown that every optimal S-alphabet of $(\mathcal{A}, \Phi)$ can be constructed in this way by finding a path which fulfills the conditions and violates the direction of the edges as seldom as possible. This proves our main theorem and answers

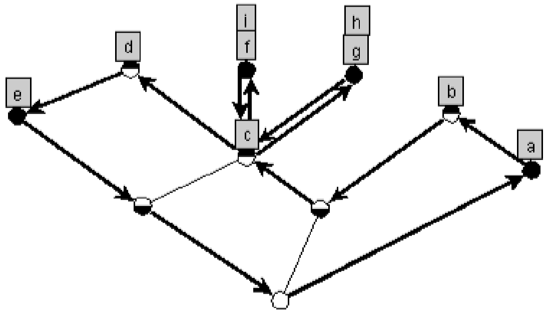

Figure 5: Hasse-diagram of example (1) with a possible path in its S-graph from which the S-alphabet $a\ b\ M_1\ c\ g\ h\ M_2\ f\ i\ M_3\ d\ M_4\ e\ M_5$ can be achieved.

questions 1 and 2.

Theorem 0.4.1 $(\mathcal{A}, \Phi)$ *is S-encodable iff the Hasse-diagram of* $(\mathcal{H}(\Phi), \supseteq)$ *is isomorphic to a plane graph* G *and for every* $b \in \mathcal{A}$ *the* b*-set lies at the boundary of the infinite face of* G'*, where* G' *is obtained from* G *by removing the vertex of the empty set and all corresponding edges.*

If $(\mathcal{A}, \Phi)$ *is S-encodable, then all optimal S-alphabets* $(\mathcal{A}, \Sigma, <)$ *of* $(\mathcal{A}, \Phi)$ *can be constructed systematically.*

0.4.2 Pāṇini's *Śivasūtras* are perfect

Figure 6 shows the Hasse-diagram, with duplicated *h*, corresponding to the *pratyāhāras* which Pāṇini uses in his *Aṣṭādhyāyī*; the duplication of h is denoted by $h\_$.[8] The 42 *pratyāhāras* actually used by Pāṇini in the *Aṣṭādhyāyī* are marked in the figure with white boxes. The black and the striped rectangles next to some of the vertices mark the places where markers have to be added, depending on the traversal direction (black: anti-clockwise [14 markers], striped: clockwise [17 markers]). It is obvious that no S-encoding can have less than 14 markers and the optimal S-alphabets are the various combinatorial variants of

⟨a,i,u,M_1, ṛ,ḷ,M_2, {⟨{e,o}, M_3⟩, ⟨{ai,au}, M_4⟩},h,y,v,r,M_5,l,M_6,
ñ,m,{n̄,ṇ,n, }, M_7, jh,bh,M_8, {gh,ḍh,dh}, M_9, j,{b,g,ḍ,d}, M_{10},
{kh,ph}, {ch,ṭh,th}, {c,ṭ,t}, M_{11}, {k,p}, M_{12}, {ś,ṣ,s}, M_{13},h,M_{14}⟩ .

Kiparsky (1991) argues in detail that the order chosen by Pāṇini out of the set of possibilities is unique if one requires a subsidiary principle of restrictiveness.

[8]The drawing was done by the tool "Concept Explorer" written by Sergey Yevtushenko, which can be found at `http://www.sourceforge.net/projects/conexp`.

So far we have answered the first three questions in the affirmative. Hence, we have argued that Pāṇini was forced to enlarge the alphabet, but it remains to show why duplicating the *h* is the best choice. If *h* is entirely removed from the *pratyāhāras*, then the optimal S-alphabet has only one marker less, namely 13.

In the *pratyāhāras*, the occurrence of *h* and any two of the segments $\{s, bh, v, l\}$ are independent of each other. Take for example the three segments h, l and v; then there exists a *pratyāhāra* – or an intersection of *pratyāhāras* – for each subset of $\{h, l, v\}$ which contains the elements of the subset but no element of its complement in $\{h, l, v\}$. Therefore, the class memberships of the segments h, l and v are independent of each other. A Hasse-diagram which contains 3 independent elements has K^5 as a minor and is therefore not planar (see figure 3). Hence, to avoid the duplication of *h* it would be necessary to duplicate at least 3 of the segments $\{s, bh, v, l\}$, which is worse than duplicating just one. For that reason, it is necessary to duplicate h in order to get a perfect S-alphabet corresponding to Pāṇini's *pratyāhāras*.

Summarizing, all 4 questions at issue have to be affirmed. Pāṇini's method of representing hierarchical information in a linear form is an interesting field of further investigations. Also the fact that one does not need to refer to phonological features explicitly in order to define phonological classes is remarkable.

Kornai (1993) points out that Pāṇini's approach is generalized by *feature geometry*, and that it is genuinely weaker than the latter. Although Kornai argues that the power of feature geometry is needed in order to get the proper set of natural classes of a phonological system, for some special tasks like describing the set of *major class features* a *Śivasūtra*-style analysis seems to be quite appropriate.

Finally, it should be emphasized again: The approach presented is so general that it is not limited to the domain of phonology.

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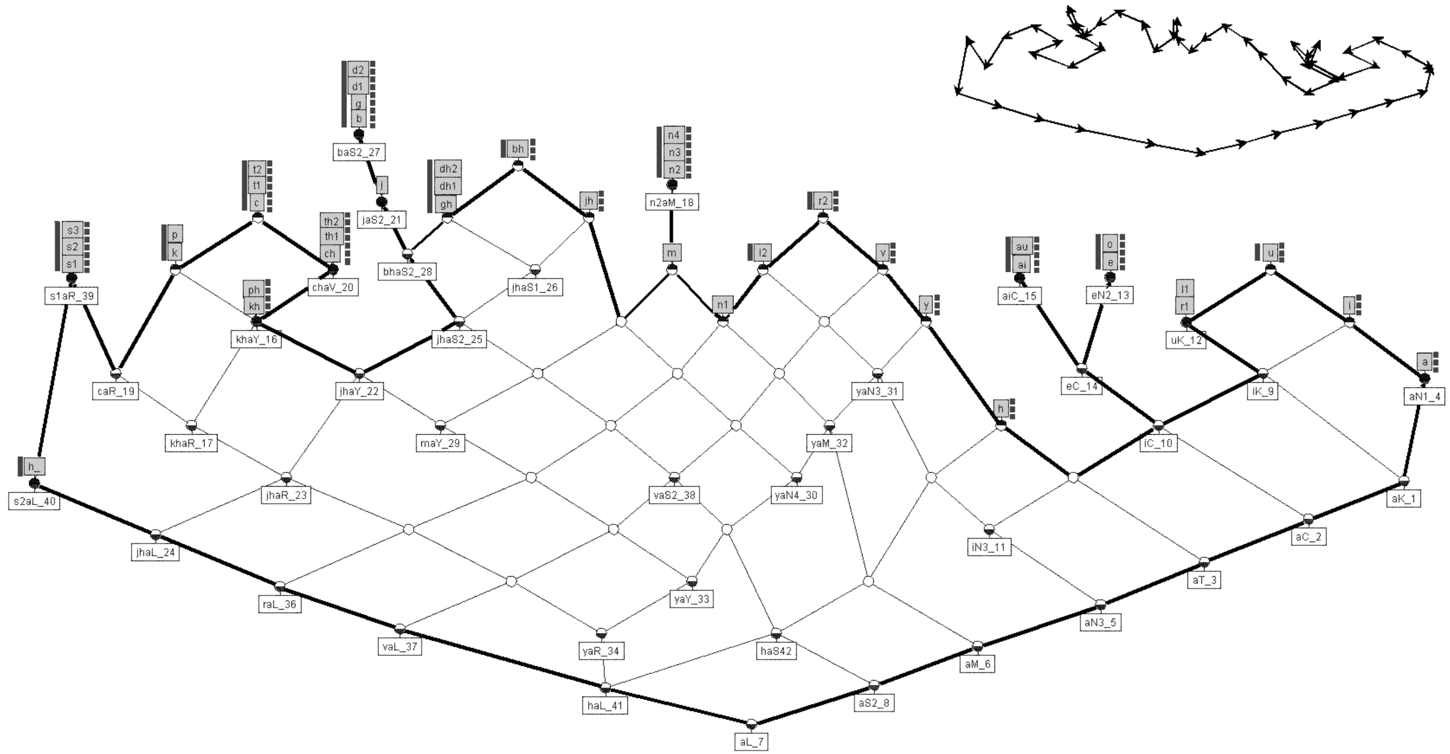

Figure 6: Hasse-diagram of the of the *pratyāhāras* used in the *Aṣṭādhyāyī.* The denotations in the figure are as follows: h_{-} is the duplicate of *h* and r1:ṛ, l1:ḷ, r2:r, l2:l, n1:ñ, n2:ṅ, n3:ṇ, n4:n, dh1:ḍh, dh2:dh, d1:ḍ, d2:d, th1:ṭh, th2:th, t1:ṭ, t2:t, s1:ś, s2:ṣ, s3:s. The white boxes mark the 42 phonological classes which Pāṇini uses in the *Aṣṭādhyāyī.* The small figure on top shows the path in the S-graph which one has to choose in order to construct Pāṇini's *Śivasūtras*.