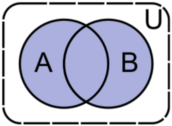
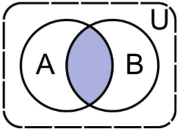
- set: a collection of entities
- a set is determined by the entities, which belong to it
- **element** ($a \in A$): an entity belongs to a set
- finite sets can be defined as a list of elements, e.g. $\{a, b, c, d, e\}$
- there is exactly one set with no elements: **empty set**, $\emptyset$
- **subset** ($A \subseteq B$): all elements of A are also elements of B
- borderline cases: $A \subseteq A$, $\emptyset \subseteq A$
- two sets A and B are equal iff $A \subseteq B$ and $B \subseteq A$
- **power set** ($\mathcal{POW}(A)$ or 2^A): the set of all subsets of A
- two sets A and B are disjoint iff $A \cap B = \emptyset$

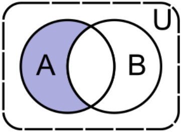
- **union** ($A \cup B$): set of entities that are in A or in B



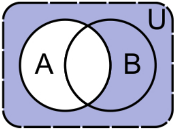
- **intersection** ($A \cap B$): set of entities that are both in A and B



- **difference** ($A \setminus B$): set of entities that are in A but not in B



- given the basic set U and set $A \subseteq U$ we call $U \setminus A$ the **complement** of A and write $\overline{A}$

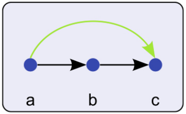


- the laws of de Morgan:
  - $\overline{A \cap B} = \overline{A} \cup \overline{B}$
  - $\overline{A \cup B} = \overline{A} \cap \overline{B}$

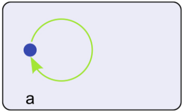
- an ***n*-tuple** is a list with $n \geq 1$ elements where the order of the elements is fixed and each element can occur any number of times
- a 2-tuple is also called an **ordered pair**
- the **Cartesian product** of n sets $A_1 \times .... \times A_n$ is the set of all n-tuples of which the i^{th} element is from the set A_i; $A_1 \times ... \times A_n := \{(x_1, ..., x_n) \mid x_i \in M_i \text{for} i = 1, ..., n\}$
  - e.g. $A = \{a, b\}$ and $B = \{c, d\}$ then $A \times B = \{(a, c), (a, d), (b, c), (b, d)\}$

  $A \times A \times ... \times A$ is also written as A^n where A occurs exactly n-times
- a subset of Cartesian products of n sets $R \subseteq A_1 \times ... \times A_n$ is called an ***n*-place relation**.
- a **binary relation** is a set of orderrd pairs
  - when a and b are in the relation R, we write $(a, b) \in R$ of aRb or $R(a, b)$ or Rab

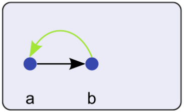
- a relation R is **transitive** if for all $a, b, c \in A$: if $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) inR$



- a relation R is **reflexive** if for all $a \in A$: $(a, a) \in R$ holds



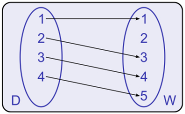
- a relation R is **symmetric** if for all $a, b \in A$: if $(a, b) \in R$ then $(b, a) \in R$



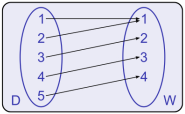
- a relation $R \subseteq A \times A$ on A if R is reflexive, symmetric and transitive
- Let R be an equivalence relation on A
  - the **equivalence class** of an element $a \in A$ is the set of all elements in A which are equivalent to a: $[a]_R = \{b \in A \mid (a, b) \in R\}$
  - the set $A/R = \{[a]_R \mid a \in A\}$ of all equivalence classes of elements of A with respect to R is called the **quotient** of A with respect to R
- Let R be an equivalence relation on A. Then it holds that
  - two equivalence class of R are either **disjoint** or **identical**:
  - for all $a, b \in A$ we have either $[a]_R \cap [b]_R = \emptyset$ or $[A]_R = [b]_R$
  - the equivalence classes of R cover the whole of A: $\bigcap A/R = A$

- a relation $R \subseteq A \times B$ is a **function** if every element of A is related to exactly one element from B
- functions must satisfy **existence** and **uniqueness**
  - existence: for all $a \in A$ there is a $b \in B$ such that $(a, b) \in R$
  - uniqueness: if $(a, b) \in R$ and $(a, c) \in R$ then $b = c$
- a relation R such that it satisfies uniqueness (but does not satisfy existence) is called a **partial function**

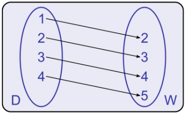
- a function f is **injektive** if two distinct elements of its domain are never related to the same element of its range: for all $a, b \in A$: if $f(a) = f(b)$ then $a = b$



- a function f is **surjective** if all for $a \in A$ there is a $b \in B$ such that $f(a) = b$



- a function f is **bijective** if f is injective and surjective



- a subset $N \subseteq A$ can be described using its characteristic function
- the **characteristic function** of a subset $N \subseteq A$ is the function $\chi : A \to \{0,1\}$ for which it holds that $\chi(x) = 1$ if and only if $x \in N$
- the characteristic function of $N \subseteq A$ is frequently written as χ_N
- it holds that

$$\chi_N : A \to \{0,1\}; \quad \chi_N(x) = \begin{cases} 1 & \text{if } x \in N \\ 0 & \text{otherwise} \end{cases}$$