# Parsing

## Probabilistic CFG (PCFG)

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Jurafsky and Martin (2009)

(Some of the slides are due to Wolfgang Maier.)

# Data-Driven Parsing

- Linguistic grammars can not only be created manually. Another way to obtain grammars is to interpret the syntactic structures in a treebank as the derivations of a latent grammar and to use an appopriate algorithm for grammar extraction.
- One can also estimate occurrence probabilities for the rules of a grammar. These can be used to determine the best parse, resp. parses of a sentence.
- Furthermore, rule probabilities can serve to speed up parsing.
- Parsing with a probabilistic grammar obtained from a treebank is called data-driven parsing.

# PCFG (1)

In most cases, probabilistic CFGs are used for data-driven parsing.

**PCFG**

A **Probabilistic Context-Free Grammar** (PCFG) is a tuple $G_P = (N, T, P, S, p)$ where $(N, T, P, S)$ is a CFG and $p : P \to [0, 1]$[a] is a function such that for all $A \in N$,

$$\sum_{A \to \alpha \in P} p(A \to \alpha) = 1$$

[a] $[0, 1]$ denotes $\{i \in \mathbb{R} \mid 0 \leq i \leq 1\}$.

$p(A \to \alpha)$ is the conditional probability $p(A \to \alpha \mid A)$

# PCFG (2)

**PCFG**

Start symbol VP

| | | | | | |
|---|---|---|---|---|---|
| | | | | 1 | Det $\rightarrow$ the |
| 0.8 | VP $\rightarrow$ V NP | 1 | PP $\rightarrow$ P NP | 1 | P $\rightarrow$ with |
| 0.2 | VP $\rightarrow$ VP PP | 0.1 | N $\rightarrow$ N PP | 0.6 | N $\rightarrow$ man |
| 1 | NP $\rightarrow$ Det N | 1 | V $\rightarrow$ sees | 0.3 | N $\rightarrow$ telescope |

# PCFG (2)

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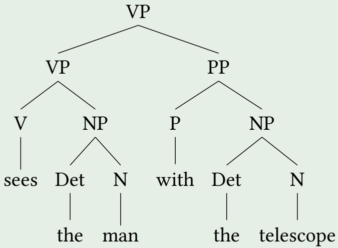
- Probability of a parse tree: product of the probabilities of the rules used to generate the parse tree.
- Probability of a category $A$ spanning a string $w$: sum of the probabilities of all parse trees with root label $A$ and yield $w$.

# PCFG (3)

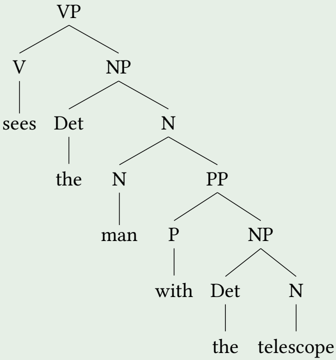
## Parse tree probability

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.8 | VP → V NP | 1 | NP → Det N | 0.1 | N → N PP | 1 | Det → the | 0.6 | N → man |
| 0.2 | VP → VP PP | 1 | PP → P NP | 1 | V → sees | 1 | P → with | 0.3 | N → telescope |

$t_1$




$t_2$

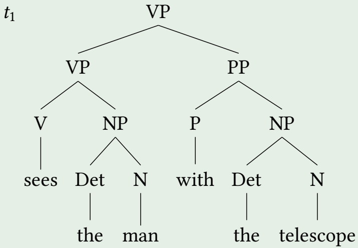


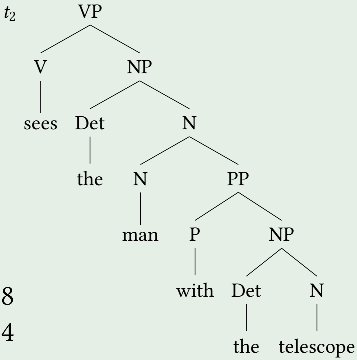

$$P(t_1) = 0.6 \cdot 0.8 \cdot 0.2 \cdot 0.3 = 0.0288$$
$$P(t_2) = 0.6 \cdot 0.8 \cdot 0.1 \cdot 0.3 = 0.0144$$

# PCFG (3)

## Parse tree probability

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.8 | VP $\to$ V NP | 1 | NP $\to$ Det N | 0.1 | N $\to$ N PP | 1 | Det $\to$ the | 0.6 | N $\to$ man |
| 0.2 | VP $\to$ VP PP | 1 | PP $\to$ P NP | 1 | V $\to$ sees | 1 | P $\to$ with | 0.3 | N $\to$ telescope |




$$P(t_1) = 0.6 \cdot 0.8 \cdot 0.2 \cdot 0.3 = 0.0288$$
$$P(t_2) = 0.6 \cdot 0.8 \cdot 0.1 \cdot 0.3 = 0.0144$$

$$p(\text{VP,sees the man with the telescope}) = 0.0288 + 0.0144$$

# PCFG (4)

Probabilities of leftmost derivations:

**Probability of a leftmost derivation**

Let $G = (N, T, P, S, p)$ be a PCFG, and let $\alpha, \gamma \in (N \cup T)^*$.

- Let $A \rightarrow \beta \in P$. The probability of a leftmost derivation $\alpha \overset{A \rightarrow \beta}{\Rightarrow}_l \gamma$ is
$$p(\alpha \overset{A \rightarrow \beta}{\Rightarrow}_l \gamma) = p(A \rightarrow \beta)$$
- Let $A_1 \rightarrow \beta_1, \ldots, A_m \rightarrow \beta_m \in P$, $m \in \mathbb{N}$. The probability of a leftmost derivation $\alpha \overset{A_1 \rightarrow \beta_1}{\Rightarrow}_l \cdots \overset{A_m \rightarrow \beta_m}{\Rightarrow}_l \gamma$ is
$$p(\alpha \overset{A_1 \rightarrow \beta_1}{\Rightarrow}_l \cdots \overset{A_m \rightarrow \beta_m}{\Rightarrow}_l \gamma) = \prod_{i=1}^{m} p(A_i \rightarrow \beta_i)$$

# PCFG (5)

- The probability of leftmost deriving $\gamma$ from $\alpha$, $\alpha \overset{*}{\Rightarrow}_l \gamma$ is defined as the sum over the probabilities of all leftmost derivations of $\gamma$ from $\alpha$:

$$p(\alpha \overset{*}{\Rightarrow}_l \gamma) = \sum_{i=1}^{k} \prod_{j=1}^{m} p(A_j^i \rightarrow \beta_j^i)$$

  where $k \in \mathbb{N}$ is the number of leftmost derivations of $\gamma$ from $\alpha$ and $m \in \mathbb{N}$ is the derivation length of the $i$th derivation and $A_j^i \rightarrow \beta_j^i$ is the $j$th derivation step of the $i$th leftmost derivation.

In the following, the subscript $l$ is omitted assuming that derivations are identified with the corresponding leftmost derivation for probabilities.

# PCFG (6)

**Consistent PCFG**

A PCFG is **consistent** if the sum of the probabilities of all sentences in the language equals 1.

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## Consistent PCFG

A PCFG is **consistent** if the sum of the probabilities of all sentences in the language equals 1.

## Example of an inconsistent PCFG

$.4\ S \rightarrow A \quad .6\ S \rightarrow B \quad 1\ A \rightarrow a \quad 1\ B \rightarrow B$

Problem: probability mass disappears into infinite derivations.

$\sum_{w \in L(G)} p(w) = p(a) = 0.4$

# PCFG (6)

## Consistent PCFG

A PCFG is **consistent** if the sum of the probabilities of all sentences in the language equals 1.

## Example of an inconsistent PCFG

$.4\ S \rightarrow A \quad .6\ S \rightarrow B \quad 1\ A \rightarrow a \quad 1\ B \rightarrow B$

Problem: probability mass disappears into infinite derivations.

$\sum_{w \in L(G)} p(w) = p(a) = 0.4$

PCFGs estimated from treebanks are usually consistent.

# Inside and outside probability (1)

Given a PCFG and an input $w = w_1 \dots w_n$, determine the likelihood of $w$, i.e., compute $\sum_{t' \in T(w)} P(t')$.

We don't want to compute the probability of every parse tree separately and then sum over the results. This is too expensive.

Instead, we adopt a computation with tabulation, in order to share the results for common subtrees.

# Inside and outside probability (2)

Idea: We fill a $|N| \times |w| \times |w|$ matrix $\alpha$ where the first dimension is the id of a non-terminal, and the second and third are the start and end indices of a span. $\alpha_{A,i,j}$ gives the probability of deriving $w_i \ldots w_j$ from $A$ or, put differently, of a parse tree with root label $A$ and yield $w_i \ldots w_j$:

$$\alpha_{A,i,j} = P(A \stackrel{*}{\Rightarrow} w_i \ldots w_j | A)$$

**Inside computation**

1. for all $1 \leq i \leq |w|$ and $A \in N$:
   if $A \rightarrow w_i \in P$, then $\alpha_{A,i,i} = p(A \rightarrow w_i)$, else $\alpha_{A,i,i} = 0$
2. for all $1 \leq i < j \leq |w|$ and $A \in N$:
   $\alpha_{A,i,j} = \sum_{A \rightarrow BC \in P} \sum_{k=i}^{j-1} p(A \rightarrow BC)\alpha_{B,i,k}\alpha_{C,k+1,j}$

We have in particular $\alpha_{S,1,|w|} = P(w)$.

# Inside and outside probability (3)

## Inside computation

0.3: S $\rightarrow$ AS  0.6: S $\rightarrow$ AX  0.1: S $\rightarrow$ a  1: X $\rightarrow$ SA  1: A $\rightarrow$ a

input $w = a^4$

| $j$ | | | | |
|---|---|---|---|---|
| 4 | | | | (1,A), (0.1,S) |
| 3 | | | (1,A), (0.1,S) | |
| 2 | | (1,A), (0.1,S) | | |
| 1 | (1,A), (0.1,S) | | | |
| | 1 | 2 | 3 | 4 $i$ |

# Inside and outside probability (3)

## Inside computation

0.3: S $\rightarrow$ AS  0.6: S $\rightarrow$ AX  0.1: S $\rightarrow$ a  1: X $\rightarrow$ SA  1: A $\rightarrow$ a

input $w = a^4$

| $j$ | | | | |
|---|---|---|---|---|
| 4 | | | ($3\cdot 10^{-2}$,S), (0.1,X) | (1,A), (0.1,S) |
| 3 | | ($3\cdot 10^{-2}$,S), (0.1,X) | (1,A), (0.1,S) | |
| 2 | ($3\cdot 10^{-2}$,S), (0.1,X) | (1,A), (0.1,S) | | |
| 1 | (1,A), (0.1,S) | | | |
| | 1 | 2 | 3 | 4 $i$ |

# Inside and outside probability (3)

## Inside computation

0.3: S $\rightarrow$ AS    0.6: S $\rightarrow$ AX    0.1: S $\rightarrow$ a    1: X $\rightarrow$ SA    1: A $\rightarrow$ a

input $w = a^4$

| $j$ | | | | |
|---|---|---|---|---|
| 4 | | ($6.9 \cdot 10^{-2}$,S), (0.03,X) | ($3 \cdot 10^{-2}$,S), (0.1,X) | (1,A), (0.1,S) |
| 3 | ($6.9 \cdot 10^{-2}$,S), (0.03,X) | ($3 \cdot 10^{-2}$,S), (0.1,X) | (1,A), (0.1,S) | |
| 2 | ($3 \cdot 10^{-2}$,S), (0.1,X) | (1,A), (0.1,S) | | |
| 1 | (1,A), (0.1,S) | | | |
| | 1 | 2 | 3 | 4 $i$ |

# Inside and outside probability (3)

## Inside computation

0.3: S $\to$ AS  0.6: S $\to$ AX  0.1: S $\to$ a  1: X $\to$ SA  1: A $\to$ a

input $w = a^4$

| $j$ | | | | |
|---|---|---|---|---|
| 4 | ($3.87 \cdot 10^{-2}$,S), (0.069,X) | ($6.9 \cdot 10^{-2}$,S), (0.03,X) | ($3 \cdot 10^{-2}$,S), (0.1,X) | (1,A), (0.1,S) |
| 3 | ($6.9 \cdot 10^{-2}$,S), (0.03,X) | ($3 \cdot 10^{-2}$,S), (0.1,X) | (1,A), (0.1,S) | |
| 2 | ($3 \cdot 10^{-2}$,S), (0.1,X) | (1,A), (0.1,S) | | |
| 1 | (1,A), (0.1,S) | | | |
| | 1 | 2 | 3 | 4 $i$ |

# Inside and outside probability (3)

## Inside computation

0.3: S $\rightarrow$ AS  0.6: S $\rightarrow$ AX  0.1: S $\rightarrow$ a  1: X $\rightarrow$ SA  1: A $\rightarrow$ a

input $w = a^4$

| $j$ | | | | |
|---|---|---|---|---|
| 4 | ($3.87 \cdot 10^{-2}$,S), (0.069,X) | ($6.9 \cdot 10^{-2}$,S), (0.03,X) | ($3 \cdot 10^{-2}$,S), (0.1,X) | (1,A), (0.1,S) |
| 3 | ($6.9 \cdot 10^{-2}$,S), (0.03,X) | ($3 \cdot 10^{-2}$,S), (0.1,X) | (1,A), (0.1,S) | |
| 2 | ($3 \cdot 10^{-2}$,S), (0.1,X) | (1,A), (0.1,S) | | |
| 1 | (1,A), (0.1,S) | | | |
| | 1 | 2 | 3 | 4 $i$ |

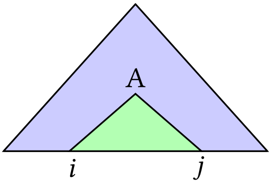
$P(aaaa) = \alpha_{S,1,4} = 0.0387$

# Inside and outside probability (4)

We can also compute the outside probability of a given non-terminal $A$ with a span from $i$ to $j$.

Inside: Sum over all possibilities for the tree below $A$ (spanning from $i$ to $j$).

Outside: Sum over all possibilities for the part of the parse tree outside the tree below $A$, i.e., over all possibilities to complete a $A$, $i$, $j$ tree into a parse tree for the entire sentence.




Outside probability $\beta_{A,i,j}$

Inside probability $\alpha_{A,i,j}$

# Inside and outside probability (5)

We fill a $|N| \times |w| \times |w|$ matrix $\beta$ such that $\beta_{A,i,j}$ gives the probability of deriving $w_1 \dots w_{i-1} A w_{j+1} \dots w_{|w|}$ from $S$ or, put differently, of deriving a tree with root label $S$ and yield $w_1 \dots w_{i-1} A w_{j+1} \dots w_{|w|}$:

$$\beta_{A,i,j} = P(S \stackrel{*}{\Rightarrow} w_1 \dots w_{i-1} A w_{j+1} \dots w_{|w|} | S)$$

We need the inside probabilities in order to compute the outside probabilities.

**Outside computation**

1. $\beta_{S,1,|w|} = 1$ and $\beta_{A,1,|w|} = 0$ for all $A \neq S$
2. for all $1 \leq i < j \leq |w|$ and $A \in N$:
   $$\beta_{A,i,j} = \sum_{B \to AC \in P} \sum_{k=j+1}^{n} p(B \to AC) \beta_{B,i,k} \alpha_{C,j+1,k} + \sum_{B \to CA \in P} \sum_{k=1}^{i-1} p(B \to CA) \beta_{B,k,j} \alpha_{C,k,i-1}$$

# Inside and outside probability (6)

## Outside computation

0.3: S $\to$ AS    0.6: S $\to$ AX    0.1: S $\to$ a    1: X $\to$ SA    1: A $\to$ a

input $w = a^3$

| | | | | |
|---|---|---|---|---|
| $j$ | | | | |
| 3 | (1,S), (0,A), (0,X) | | | |
| 2 | | | | |
| 1 | | | | |
| | 1 | 2 | 3 | $i$ |

# Inside and outside probability (6)

## Outside computation

0.3: S $\rightarrow$ AS    0.6: S $\rightarrow$ AX    0.1: S $\rightarrow$ a    1: X $\rightarrow$ SA    1: A $\rightarrow$ a

input $w = a^3$

| $j$ | | | | |
|---|---|---|---|---|
| 3 | (1,S), (0,A), (0,X) | (0.3,S), (0,A), (0.6,X) | | |
| 2 | (0,S), (0,X), (0.03,A) | | | |
| 1 | | | | |
| | 1 | 2 | 3 | $i$ |

# Inside and outside probability (6)

## Outside computation

0.3: S $\rightarrow$ AS   0.6: S $\rightarrow$ AX   0.1: S $\rightarrow$ a   1: X $\rightarrow$ SA   1: A $\rightarrow$ a

input $w = a^3$

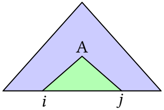
| $j$ | | | | |
|---|---|---|---|---|
| 3 | (1,S), (0,A), (0,X) | (0.3,S), (0,A), (0.6,X) | ($9 \cdot 10^{-2}$,S), (0.18,X), ($3 \cdot 10^{-2}$,A) | |
| 2 | (0,S), (0,X), (0.03,A) | (0.6,S), (0,X), ($8.99 \cdot 10^{-3}$,A) | | |
| 1 | (0,S), (0,X), ($6.9 \cdot 10^{-2}$,A) | | | |
| | 1 | 2 | 3 | $i$ |

# Inside and outside probability (7)

The following holds:

1. The probability of a parse tree for $w$ with a node labeled $A$ that spans $w_i \dots w_j$ is

$$P(S \stackrel{*}{\Rightarrow} w_1 \dots w_{i-1} A w_{j+1} \dots w_n \stackrel{*}{\Rightarrow} w_1 \dots w_n) = \alpha_{A,i,j} \beta_{A,i,j}$$




2. In particular: $P(w) = \alpha_{S,1,|w|}$

# Parsing (1)

- In PCFG parsing, we want to compute the most probable parse tree (= most probable (leftmost) derivation) given an input sentence $w$, also called the **Viterbi** parse.
- This means that we are disambiguating: Among several readings, we search for the best.
- Sometimes, the $k$ best are searched for ($k > 1$).
- During parsing, we must make sure that updates on probabilities (because a better derivation has been found for a non-terminal) do not require updates on other parts of the chart. ⇒ the order should be such that an item is used within a derivation only when its final probability is reached.

# Parsing (2)

We can extend the symbolic CYK parser to a probabilistic one. Instead of summing over all derivations (as in the computation of the inside probability), we keep the best one (⇒ **Viterbi algorithm**).

Assume a three-dimensional chart $C$ (non-terminal, start index, length).

$C_{A,i,l} := 0$ `for all` $A, i, l$
$C_{A,i,1} := p$ `if` $p : A \to w_i \in P$ `scan`
`for all` $l \in [1..n]$:
  `for all` $i \in [1..n-l+1]$:
    `for every` $p : A \to B\ C$:
      `for every` $l_1 \in [1..l-1]$:
        $C_{A,i,l} = max\{C_{A,i,l}, p \cdot C_{B,i,l_1} \cdot C_{C,i+l_1,l-l_1}\}$ `complete`

# Parsing (3)

We extend this to a parser.

- The parser can also deal with unary productions $A \rightarrow B$.
- Every chart field has three components, the probability, the rule that has been used and, if the rule is binary, the length $l_1$ of the first righthand side element.
- We assume that the grammar does not contain any loops $A \stackrel{+}{\Rightarrow} A$.

# Parsing (4)

```
C_{A,i,1} = ⟨p, A → w_i, −⟩  if  p : A → w_i ∈ P                    scan
for all l ∈ [1..n] and for all i ∈ [1..n − l]:
  for all p : A → B C and for all l_1 ∈ [1..l − 1]:
    for all l_1 ∈ [1..l − 1]:
      if C_{B,i,l_1} ≠ ∅ and C_{C,i+l_1,l−l_1} ≠ ∅ then:
        p_new = p · C_{B,i,l_1}[1] · C_{C,i+l_1,l−l_1}[1]
        if C_{A,i,l} == ∅ or C_{A,i,l}[1] < p_new then:
          C_{A,i,l} = ⟨p_new, A → BC, l_1⟩              binary complete
  repeat until C does not change any more:
    for every p : A → B:
      if C_{B,i,l} ≠ ∅ then:
        p_new = p · C_{B,i,l}[1]
        if C_{A,i,l} == ∅ or C_{A,i,l}[1] < p_new then:
          C_{A,i,l} = ⟨p_new, A → B, −⟩                 unary complete
return build_tree(S, 1, n)
```

# Parsing (5)

## Example

| | | | | | |
|---|---|---|---|---|---|
| .1 | VP $\to$ VP NP | 1 | NP $\to$ Det N | .3 | V $\to$ eats |
| .6 | VP $\to$ V NP | .3 | V $\to$ sees | 1 | Det $\to$ this |
| .3 | VP $\to$ V | .4 | V $\to$ comes | .5 | N $\to$ morning |
| .5 | N $\to$ apple | | | | |

Start symbol VP, input $w =$ *eats this morning*

| $l$ | | | | |
|---|---|---|---|---|
| 3 | | | | |
| 2 | | | | |
| 1 | | | | |
| | 1 | 2 | 3 | $i$ |

# Parsing (5)

## Example

| | | | | | |
|---|---|---|---|---|---|
| .1 | $VP \rightarrow VP\ NP$ | 1 | $NP \rightarrow Det\ N$ | .3 | $V \rightarrow$ eats |
| .6 | $VP \rightarrow V\ NP$ | .3 | $V \rightarrow$ sees | 1 | $Det \rightarrow$ this |
| .3 | $VP \rightarrow V$ | .4 | $V \rightarrow$ comes | .5 | $N \rightarrow$ morning |
| .5 | $N \rightarrow$ apple | | | | |

Start symbol VP, input $w =$ *eats this morning*

| $l$ | | | | |
|---|---|---|---|---|
| 3 | | | | |
| 2 | | | | |
| 1 | .3, $V \rightarrow$ eats | 1, $Det \rightarrow$ this | .5, $N \rightarrow$ morning | |
| | 1 | 2 | 3 | $i$ |

# Parsing (5)

## Example

| | | | | | |
|---|---|---|---|---|---|
| .1 | VP → VP NP | 1 | NP → Det N | .3 | V → eats |
| .6 | VP → V NP | .3 | V → sees | 1 | Det → this |
| .3 | VP → V | .4 | V → comes | .5 | N → morning |
| .5 | N → apple | | | | |

Start symbol VP, input $w =$ *eats this morning*

| $l$ | | | |
|---|---|---|---|
| 3 | | | |
| 2 | | | |
| 1 | .09, VP → V<br>.3, V → eats | 1, Det → this | .5, N → morning |
| | 1 | 2 | 3 |

$i$

# Parsing (5)

## Example

| | | | | | |
|---|---|---|---|---|---|
| .1 | VP $\to$ VP NP | 1 | NP $\to$ Det N | .3 | V $\to$ eats |
| .6 | VP $\to$ V NP | .3 | V $\to$ sees | 1 | Det $\to$ this |
| .3 | VP $\to$ V | .4 | V $\to$ comes | .5 | N $\to$ morning |
| .5 | N $\to$ apple | | | | |

Start symbol VP, input $w =$ *eats this morning*

| $l$ | | | |
|---|---|---|---|
| 3 | | | |
| 2 | | .5, NP $\to$ Det N, 1 | |
| 1 | .09, VP $\to$ V<br>.3, V $\to$ eats | 1, Det $\to$ this | .5, N $\to$ morning |
| | 1 | 2 | 3 | $i$ |

# Parsing (5)

## Example

| | | | | | |
|---|---|---|---|---|---|
| .1 | VP $\to$ VP NP | 1 | NP $\to$ Det N | .3 | V $\to$ eats |
| .6 | VP $\to$ V NP | .3 | V $\to$ sees | 1 | Det $\to$ this |
| .3 | VP $\to$ V | .4 | V $\to$ comes | .5 | N $\to$ morning |
| .5 | N $\to$ apple | | | | |

Start symbol VP, input $w =$ *eats this morning*

| $l$ | | | | |
|---|---|---|---|---|
| 3 | .0045, VP $\to$ VP NP, 1 | | | |
| 2 | | .5, NP $\to$ Det N, 1 | | |
| 1 | .09, VP $\to$ V<br>.3, V $\to$ eats | 1, Det $\to$ this | .5, N $\to$ morning | |
| | 1 | 2 | 3 | $i$ |

# Parsing (5)

## Example

| | | | | | |
|---|---|---|---|---|---|
| .1 | $VP \to VP\ NP$ | 1 | $NP \to Det\ N$ | .3 | $V \to$ eats |
| .6 | $VP \to V\ NP$ | .3 | $V \to$ sees | 1 | $Det \to$ this |
| .3 | $VP \to V$ | .4 | $V \to$ comes | .5 | $N \to$ morning |
| .5 | $N \to$ apple | | | | |

Start symbol VP, input $w =$ *eats this morning*

| $l$ | | | | |
|---|---|---|---|---|
| 3 | .09, $VP \to V\ NP$, 1 | | | |
| 2 | | .5, $NP \to Det\ N$, 1 | | |
| 1 | .09, $VP \to V$<br>.3, $V \to$ eats | 1, $Det \to$ this | .5, $N \to$ morning | |
| | 1 | 2 | 3 | $i$ |

(The analysis of the VP gets revised since a better parse tree has been found.)

Jurafsky, D. and Martin, J. H. (2009). *Speech and Language Processing. An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition.* Prentice Hall Series in Articial Intelligence. Pearson Education International, second edition edition.