

# Machine Learning Exercises: PCFG 1

Laura Kallmeyer

Summer 2016, Heinrich-Heine-Universität Düsseldorf

**Exercise 1** Consider the following PCFG  $G = \langle \{S, A, X\}, \{a\}, P, S, p \rangle$  (see course slides) with  $P$  and  $p$  as follows:

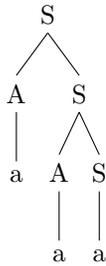
$$0.3: S \rightarrow AS \quad 0.6: S \rightarrow AX \quad 0.1: S \rightarrow a \quad 1: X \rightarrow SA \quad 1: A \rightarrow a$$

Given this PCFG,

1. give the different parse trees for  $w = aaa$  with their sets of productions with span indices and with their probabilities.
2. what is the best parse tree for  $w = aaa$ ?
3. what is the probability of  $aaa$ ?

Solution:

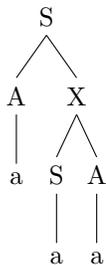
1.  $t_1$ :



$$\begin{aligned}
 [S, 1, 3] &\rightarrow [A, 1, 1][S, 2, 3] \\
 [S, 2, 3] &\rightarrow [A, 2, 2][S, 3, 3] \\
 [A, 1, 1] &\rightarrow a, [A, 2, 2] \rightarrow a, [S, 3, 3] \rightarrow a
 \end{aligned}$$

$$p(t_1) = 0.3 \cdot 1 \cdot 0.3 \cdot 1 \cdot 1 \cdot 0.1 = 0.009$$

$t_2$ :



$$\begin{aligned}
 [S, 1, 3] &\rightarrow [A, 1, 1][X, 2, 3] \\
 [X, 2, 3] &\rightarrow [S, 2, 2][A, 3, 3] \\
 [A, 1, 1] &\rightarrow a, [S, 2, 2] \rightarrow a, [A, 3, 3] \rightarrow a
 \end{aligned}$$

$$p(t_2) = 0.6 \cdot 1 \cdot 1 \cdot 0.1 \cdot 1 \cdot 1 = 0.06$$

2.  $t_2$

3.  $0.009 + 0.06 = 0.069$  (see also field  $\alpha_{S,1,3}$  in the inside matrix on slide 12)