Machine Learning
Exercises: HMM

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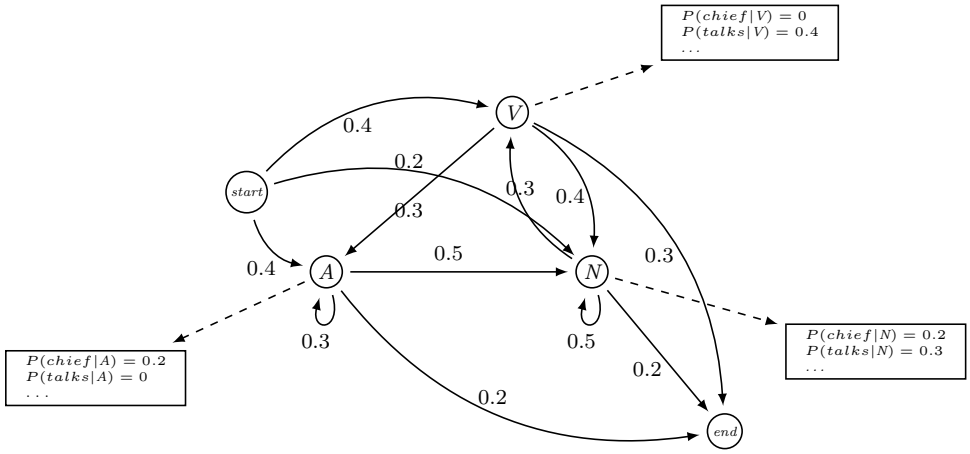
Exercise 1 *Consider the following HMM for POS tagging:*



1. *Given this HMM, calculate the forward and backward probabilities α and β for the observation sequence "chief talks".*
2. *What is the probability of this sequence? How can this probability be obtained from the α and β tables?*

Solution:

1. α:

V	0	$4.79 \cdot 10^{-3}$
N	$4 \cdot 10^{-2}$	$1.8 \cdot 10^{-2}$
A	$8 \cdot 10^{-2}$	0
t	1	2

β:

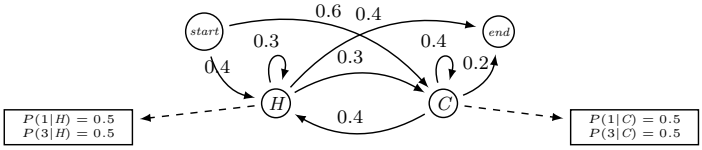
V	$2.4 \cdot 10^{-2}$	0.3
N	$6.6 \cdot 10^{-2}$	0.2
A	$3 \cdot 10^{-2}$	0.2
t	1	2

2. $0 \cdot 2.4 \cdot 10^{-2} + 4 \cdot 10^{-2} \cdot 6.6 \cdot 10^{-2} + 8 \cdot 10^{-2} \cdot 3 \cdot 10^{-2} = 5.02 \cdot 10^{-3}$

   or

   $4.79 \cdot 10^{-3} \cdot 0.3 + 1.8 \cdot 10^{-2} \cdot 0.2 + 0 \cdot 0.2 = 5.02 \cdot 10^{-3}$

Exercise 2 *Now consider again the ice cream example from the course slides:*



assume that the observed sequence is 31*.*

The forward and backward matrices for this input are:

α:

H	0.2	$9 \cdot 10^{-2}$
C	0.3	$9 \cdot 10^{-2}$
t	*1*	*2*

β:

H	$9 \cdot 10^{-2}$	0.4
C	0.12	0.2
t	*1*	*2*

$P(31) = 5.4 \cdot 10^{-2}$

Calculate one iteration of the forward-backward EM algorithm in order to estimate new probabilities.

Solution:

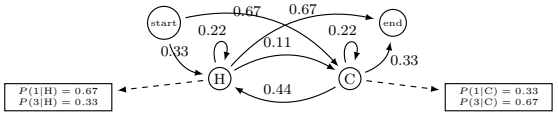
E-step: γ:

t	H	C
1	0.33	0.67
2	0.67	0.33

ξ_1:

	j =H	j =C
i =H	0.22	0.11
i =C	0.44	0.22

M-step:



$P(31) = 0.11$

Further steps (were not asked in the exercise):

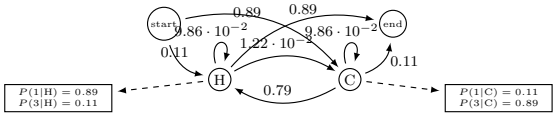
E-step: γ:

t	H	C
1	0.11	0.89
2	0.89	0.11

ξ_1:

	j =H	j =C
i =H	$9.86 \cdot 10^{-2}$	$1.22 \cdot 10^{-2}$
i =C	0.79	$9.86 \cdot 10^{-2}$

M-step:



$P(31) = 0.5$

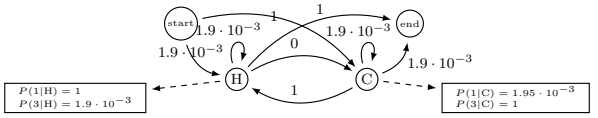
E-step: γ:

t	H	C
1	$1.9 \cdot 10^{-3}$	1
2	1	$1.9 \cdot 10^{-3}$

ξ_1:

	j =H	j =C
i =H	$1.9 \cdot 10^{-3}$	0
i =C	1	$1.9 \cdot 10^{-3}$

M-step:



$P(31) = 0.99$

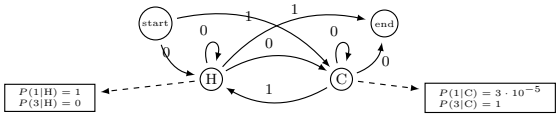
E-step: γ:

t	H	C
1	0	1
2	1	0

ξ_1:

	j =H	j =C
i =H	0	0
i =C	1	0

M-step:



$P(31) = 1$