Mildly Context-Sensitive Grammar Formalisms:

Tree Adjoining Grammar Parsing

Laura Kallmeyer
Heinrich-Heine-Universität Düsseldorf

Sommersemester 2011

Overview

1. A CYK recognizer for TAG
   (a) Items
   (b) Inference rules
   (c) Complexity
2. Earley Parsing for TAG
   (a) Introduction
   (b) Items
   (c) Inference rules

CYK: Items (1)

CYK-Parsing for TAG:

- First presented in [Vijay-Shanker and Joshi, 1985], formulation with deduction rules in [Kallmeyer and Satta, 2009].
- Assumption: elementary trees are such that each node has at most two daughters. (Any TAG can be transformed into an equivalent TAG satisfying this condition.)
- The algorithm simulates a bottom-up traversal of the derived tree.

CYK: Items (2)

- At each moment, we are in a specific node in an elementary tree and we know about the yield of the part below. Either there is a foot node below, then the yield is separated into two parts. Or there is no foot node below and the yield is a single substring of the input.
- We need to keep track of whether we have already adjoined at the node or not since at most one adjunction per node can occur. For this, we distinguish between a bottom and a top position for the dot on a node. Bottom signifies that we have not performed an adjunction.

CYK: Items (3)

Item form: $[\gamma, p_t, i, f_1, f_2, j]$ where

- $\gamma \in I \cup A$,
- p is the Gorn address of a node in γ (ϵ for the root, pi for the ith daughter of the node at address p),
- subscript $t \in \{\top, \bot\}$ specifies whether substitution or adjunction has already taken place ($\top$) or not ($\bot$) at p, and
- $0 \leq i \leq f_1 \leq f_2 \leq j \leq n$ are indices with i, j indicating the left and right boundaries of the yield of the subtree at position p and f_1, f_2 indicating the yield of a gap in case a foot node is dominated by p. We write $f_1 = f_2 = -$ if no gap is involved.

CYK: Inference rules (1)

Goal items: $[\alpha, \epsilon_\top, 0, -, -, n]$ where $\alpha \in I$

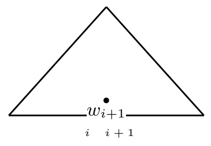
We need two rules to process leaf nodes while scanning their labels, depending on whether they have terminal labels or labels ϵ:

Lex-scan: $$\frac{}{[\gamma, p_\top, i, -, -, i+1]} \; l(\gamma, p) = w_{i+1}$$

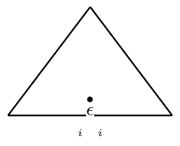
Eps-scan: $$\frac{}{[\gamma, p_\top, i, -, -, i]} \; l(\gamma, p) = \epsilon$$

(Notation: $l(\gamma, p)$ is the label of the node at address p in γ.)

CYK: Inference rules (2)



Lex-scan

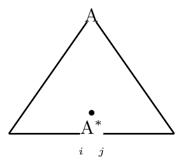


Eps-scan

CYK: Inference rules (3)

The rule **foot-predict** processes the foot node of auxiliary trees $\beta \in A$ by guessing the yield below the foot node:

Foot-predict: $$\frac{}{[\beta, p_\top, i, i, j, j]} \; \beta \in A, p \text{ foot node address in } \beta, i \leq j$$



CYK: Inference rules (4)

When moving up inside a single elementary tree, we either move from only one daughter to its mother, if this is the only daughter, or we move from the set of both daughters to the mother node:

Move-unary:

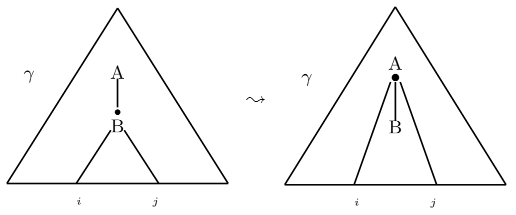
$$\frac{[\gamma, (p \cdot 1)_\top, i, f_1, f_2, j]}{[\gamma, p_\bot, i, f_1, f_2, j]} \quad \text{node address } p \cdot 2 \text{ does not exist in } \gamma$$

Move-binary: $\dfrac{[\gamma, (p \cdot 1)_\top, i, f_1, f_2, k], [\gamma, (p \cdot 2)_\top, k, f_1', f_2', j]}{[\gamma, p_\bot, i, f_1 \oplus f_1', f_2 \oplus f_2', j]}$

($f' \oplus f'' = f$ where $f = f'$ if $f'' = -$, $f = f''$ if $f' = -$, and f is undefined otherwise)

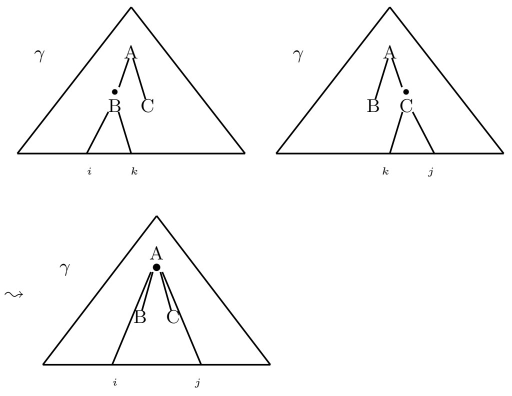
CYK: Inference rules (5)

Move-unary:



CYK: Inference rules (6)

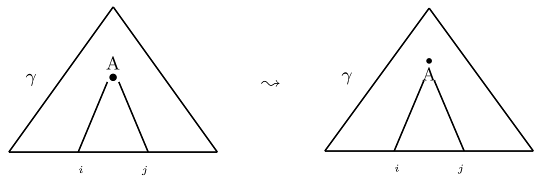
Move-binary:



CYK: Inference rules (7)

For nodes that do not require adjunction, we can move from the bottom position of the node to its top position.

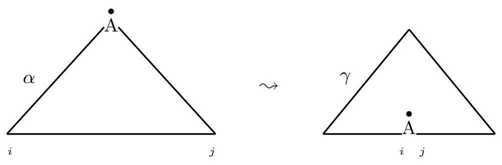
Null-adjoin: $\dfrac{[\gamma, p_\bot, i, f_1, f_2, j]}{[\gamma, p_\top, i, f_1, f_2, j]}$ $f_{OA}(\gamma, p) = 0$



CYK: Inference rules (8)

The rule **substitute** performes a substitution:

Substitute: $\dfrac{[\alpha, \epsilon_\top, i, -, -, j]}{[\gamma, p_\top, i, -, -, j]}$ $l(\alpha, \epsilon) = l(\gamma, p)$



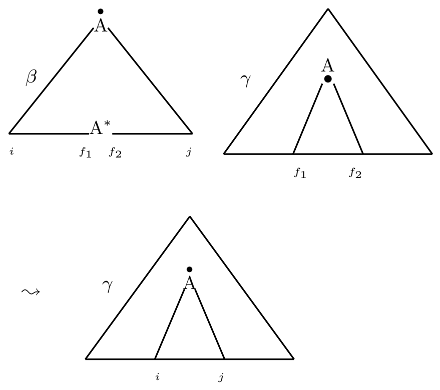
CYK: Inference rules (9)

The rule **adjoin** adjoins an auxiliary tree β at p in γ, under the precondition that the adjunction of β at p in γ is allowed:

Adjoin: $\dfrac{[\beta, \epsilon_\top, i, f_1, f_2, j], [\gamma, p_\bot, f_1, f_1', f_2', f_2]}{[\gamma, p_\top, i, f_1', f_2', j]}$ $\beta \in f_{SA}(\gamma, p)$

CYK: Inference rules (10)

Adjoin:



CYK: Complexity

Complexity of the algorithm: What is the upper bound for the number of applications of the **adjoin** operation?

- We have $|A|$ possibilities for β, $|A \cup I|$ for γ, m for p where m is the maximal number of internal nodes in an elementary tree.
- The six indices $i, f_1, f_1', f_2', f_2, j$ range from 0 to n.

Consequently, **adjoin** can be applied at most $|A||A \cup I|m(n+1)^6$ times and therefore, the time complexity of this algorithm is $\mathcal{O}(n^6)$.

Earley: Introduction (1)

- Left-to-right CYK parser very slow: $O(n^6)$ worst case **and** best case (just as in CFG version of CYK, to many partial trees not pertinent to the final tree are produced).
- Behaviour is due to pure bottom-up approach, no predictive information whatsoever is used.
- Goal: Earley-style parser! First in [Schabes and Joshi, 1988]. Here, we present the algorithm from [Joshi and Schabes, 1997].

We assume a TAG without substitution nodes.

Earley: Introduction (2)

- Earley Parsing: Left-to-right scanning of the string (using predictions to restrict hypothesis space)
- Traversal of elementary trees, current position marked with a dot.

  The dot can have exactly four positions with respect to the node: left above (la), left below (lb), right above (ra), right below (rb).

Earley: Introduction (3)

General idea: Whenever we are

- left above a node, we can predict an adjunction and start the traversal of the adjoined tree;
- left of a foot node, we can move back to the adjunction site and traverse the tree below it;
- right of an adjunction site, we continue the traversal of the adjoined tree at the right of its foot node;
- right above the root of an auxiliary tree, we can move back to the right of the adjunction site.

Earley: Items (1)

What kind of information do we need in an item characterizing a partial parsing result?

$$[\alpha, dot, pos, i, j, k, l, sat?]$$

where

- $\alpha \in I \cup A$ is a (dotted) tree, *dot* and *pos* the address and location of the dot
- i, j, k, l are indices on the input string, where $i, l \in \{0, \dots, n\}$, $j, k \in \{0, \dots, n\} \cup \{-\}$, $n = |w|$, $-$ means unbound value
- *sat*? is a flag. It controls (prevents) multiple adjunctions at a single node ($sat? = 1$ means that something has already been adjoined to the dotted node)

Earley: Items (2)

What do the items mean?

- $[\alpha, dot, la, i, j, k, l, 0]$: In α part left of the dot ranges from i to l. If α is an auxiliary tree, part below foot node ranges from j to k.
- $[\alpha, dot, lb, i, -, -, i, 0]$: In α part below dotted node starts at position i.
- $[\alpha, dot, rb, i, j, k, l, sat?]$: In α part below dotted node ranges from i to l. If α is an auxiliary tree, part below foot node ranges from j to k. If $sat? = 0$, nothing was adjoined to dotted node, $sat? = 1$ means that adjunction took place.
- $[\alpha, dot, ra, i, j, k, l, 0]$: In α part left and below dotted node ranges from i to l. If α is an auxiliary tree, part below foot node ranges from j to k.

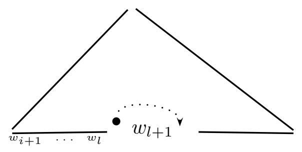
Earley: Items (3)

Some notational conventions:

- We use Gorn addresses for the nodes: 0 is the address of the root, i $(1 \leq i)$ is the address of the ith daughter of the root, and for $p \neq 0$, $p \cdot i$ is the address of the ith daughter of the node at address p.
- For a tree α and a Gorn address dot, $\alpha(dot)$ denotes the node at address dot in α (if defined).

Earley: Inference Rules (1)

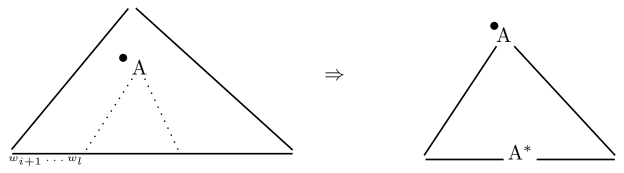
ScanTerm $\dfrac{[\gamma, dot, la, i, j, k, l, 0]}{[\gamma, dot, ra, i, j, k, l+1, 0]}$ $l(\gamma, dot) = w_{l+1}$



Scan-ε $\dfrac{[\gamma, dot, la, i, j, k, l, 0]}{[\gamma, dot, ra, i, j, k, l, 0]}$ $l(\gamma, dot) = \varepsilon$

Earley: Inference Rules (2)

PredictAdjoinable $\dfrac{[\gamma, dot, la, i, j, k, l, 0]}{[\beta, 0, la, l, -, -, l, 0]}$ $\beta \in f_{SA}(\gamma, dot)$

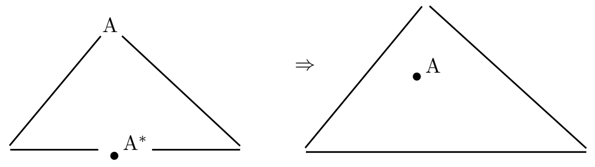


PredictNoAdj $\dfrac{[\gamma, dot, la, i, j, k, l, 0]}{[\gamma, dot, lb, l, -, -, l, 0]}$ $f_{OA}(\gamma, dot) = 0$

Earley: Inference Rules (3)

PredictAdjoined

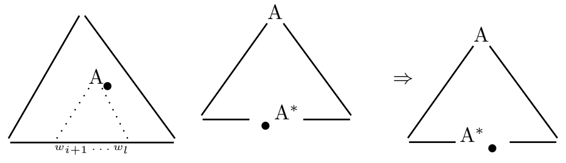
$$\frac{[\beta, dot, lb, l, -, -, l, 0]}{[\gamma, dot', lb, l, -, -, l, 0]} \quad dot = foot(\beta), \beta \in f_{SA}(\gamma, dot')$$



Earley: Inference Rules (4)

CompleteFoot

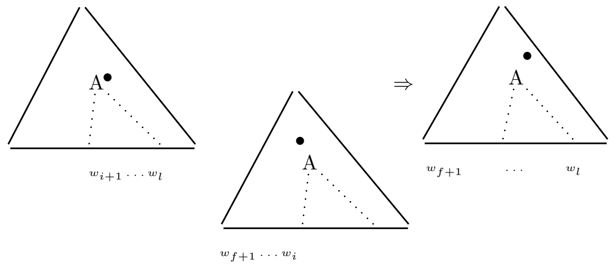
$$\frac{[\gamma, dot, rb, i, j, k, l, 0], [\beta, dot', lb, i, -, -, i, 0]}{[\beta, dot', rb, i, i, l, l, 0]} \quad {\scriptstyle dot' = foot(\beta), \atop \beta \in f_{SA}(\gamma, dot')}$$



Earley: Inference Rules (5)

CompleteNode

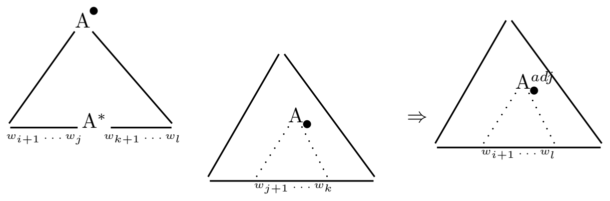
$$\frac{[\gamma, dot, la, f, g, h, i, 0], [\gamma, dot, rb, i, j, k, l, sat?]}{[\gamma, dot, ra, f, g \oplus j, h \oplus k, l, 0]} \quad l(\beta, dot) \in N$$



Earley: Inference Rules (6)

Adjoin

$$\frac{[\beta, \varepsilon, ra, i, j, k, l, 0], [\gamma, dot, rb, j, p, q, k, 0]}{[\gamma, dot, rb, i, p, q, l, 1]} \quad \beta \in f_{SA}(\gamma, p)$$



sat? = 1 prevents the new item from being reused in another Adjoin application.

Earley: Inference Rules (7)

Move the dot to daughter/sister/mother:

MoveDown: $$\frac{[\gamma, dot, lb, i, j, k, l, 0]}{[\gamma, dot\cdot 1, la, i, j, k, l, 0]} \ \gamma(dot \cdot 1) \text{ is defined}$$

MoveRight: $$\frac{[\gamma, dot, ra, i, j, k, l, 0]}{[\gamma, dot + 1, la, i, j, k, l, 0]} \ \gamma(dot + 1) \text{ is defined}$$

MoveUp: $$\frac{[\gamma, dot\cdot m, ra, i, j, k, l, 0]}{[\gamma, dot, rb, i, j, k, l, 0]} \ \gamma(dot \cdot m + 1) \text{ is not defined}$$

Earley: Inference Rules (8)

Initialize: $$\frac{}{[\alpha, \varepsilon, la, 0, -, -, 0, 0]} \ \alpha \in I, l(\alpha, \varepsilon) = S$$

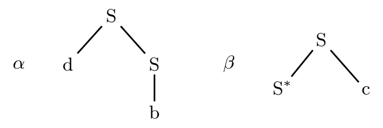
Goal item: $[\alpha, 0, ra, 0, -, -, n, 0]$, $\alpha \in I$

Earley: The Valid Prefix Property (VPP) (1)

- The Earley algorithm, as presented, does not have the VPP.
- In other words, there are items which are not part of a derivation from an initial α with the span of the derived tree up to the dotted node being a prefix of a word in the language.

Earley: The Valid Prefix Property (VPP) (2)

Example:



Every word in the language starts with *d*.

Earley: The Valid Prefix Property (VPP) (3)

Input *bccc* leads (among others) to the following items:

	Item	Rule
1.	$[\alpha, \varepsilon, la, 0, -, -, 0, 0]$	initialize
2.	$[\beta, \varepsilon, la, 0, -, -, 0, 0]$	predictAdjoinable from 1.
	...	
3.	$[\beta, 1, lb, 0, -, -, 0, 0]$	
4.	$[\alpha, 2, lb, 0, -, -, 0, 0]$	predictAdjoined from 3.
	...	
5.	$[\alpha, 2, rb, 0, -, -, 1, 0]$	
6.	$[\beta, 1, rb, 0, 0, 1, 1, 0]$	completeFoot form 3. and 5.
	...	
7.	$[\beta, \varepsilon, ra, 0, 0, 3, 4, 0]$	(after repeated adjunctions of β)
8.	$[\alpha, 2, rb, 0, -, -, 4, 1]$	adjoin from 7. and 4.

Earley: The Valid Prefix Property (VPP) (4)

- Reason for lack of VPP: neither **predictAdjoined** nor **completeFoot** nor **adjoin** check for the existence of an item that has triggered the prediction of this adjunction.
- Maintaining the VPP leads to deduction rules with more indices. It was therefore considered to be costly: $\mathcal{O}(n^9)$ [Schabes and Joshi, 1988].
- But: in some rules, some of the indices are not relevant for the rule and can be factored out (treated as "don't care"-values). Therefore, a $\mathcal{O}(n^6)$ VPP Earley algorithm is actually possible [Nederhof, 1997].

References

[Joshi and Schabes, 1997] Joshi, A. K. and Schabes, Y. (1997). Tree-Adjoining Grammars. In Rozenberg, G. and Salomaa, A., editors, *Handbook of Formal Languages*, pages 69–123. Springer, Berlin.

[Kallmeyer and Satta, 2009] Kallmeyer, L. and Satta, G. (2009). A polynomial-time parsing algorithm for tt-mctag. In *Proceedings of ACL*, Singapore.

[Nederhof, 1997] Nederhof, M.-J. (1997). Solving the correct-prefix property for TAGs. In Becker, T. and Krieger, H.-U., editors, *Proceedings of the Fifth Meeting on Mathematics of Language*, pages 124–130, Schloss Dagstuhl, Saarbrücken.

[Schabes and Joshi, 1988] Schabes, Y. and Joshi, A. K. (1988). An Earley-type parsing algorithm for Tree Adjoining Grammars. In

Proceedings of the 26th Annual Meeting of the Association for Computational Linguistics, pages 258–269.

[Vijay-Shanker and Joshi, 1985] Vijay-Shanker, K. and Joshi, A. K. (1985). Some computational properties of Tree Adjoining Grammars. In *Proceedings of the 23rd Annual Meeting of the Association for Computational Linguistics*, pages 82–93.