Mildly Context-Sensitive Grammar Formalisms:

Introduction

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Overview

1. CFG and natural languages
2. Polynomial extensions of CFG
3. Basic definitions

CFG and natural languages (1)

A context-free grammar (CFG) is a set of rewriting rules that tell us how to replace a non-terminal by a sequence of non-terminal and terminal symbols.

Example:

$S \rightarrow a\ S\ b \quad S \rightarrow ab$

The string language generated by this grammar is $\{a^n b^n \mid n \geq 1\}$.

CFG and natural languages (2)

Sample CFG $G_{telescope}$:

S	→	NP VP	NP	→	D N
VP	→	VP PP | V NP	N	→	N PP
PP	→	P NP			
N	→	man | girl | telescope	D	→	the
N	→	John	P	→	with
V	→	saw			

CFG and natural languages (3)

Context-free languages (CFLs)

- can be recognized in polynomial time ($\mathcal{O}(n^3)$);
- are accepted by push-down automata;
- have nice closure properties (e.g., closure under homomorphisms, intersection with regular languages ...);
- satisfy a pumping lemma;
- can describe nested dependencies ($\{ww^R \mid w \in T^*\}$).

(Hopcroft and Ullman, 1979)

CFG and natural languages (4)

Question: Is CFG powerful enough to describe all natural language phenomea?

Answer: No. There are constructions in natural languages that cannot be adequately described with a context-free grammar.

Example: cross-serial dependencies in Dutch and in Swiss German.

Dutch:

(1)

... dat Wim Jan Marie de kinderen zag helpen leren zwemmen

... that Wim Jan Marie the children saw help teach swim

'... that Wim saw Jan help Marie teach the children to swim'

CFG and natural languages (5)

Swiss German:

(2)

... das mer em Hans es huus hälfed aastriiche

... that we Hans$_{Dat}$ house$_{Acc}$ helped paint

'... that we helped Hans paint the house'

(3)

... das mer d'chind em Hans es huus lönd hälfe aastriiche

... that we the children$_{Acc}$ Hans$_{Dat}$ house$_{Acc}$ let help paint

'... that we let the children help Hans paint the house'

Swiss German uses case marking and displays cross-serial dependencies.

(Shieber, 1985) shows that Swiss German is not context-free.

CFG and natural languages (6)

In general, because of the closure properties, the following holds:

A formalism that can generate cross-serial dependencies can also generate the copy language $\{ww \mid w \in \{a, b\}^*\}$.

The copy language is not context-free.

Therefore we are interested in extensions of CFG in order to describe all natural language phenomena.

CFG and natural languages (7)

Idea (Joshi, 1985): characterize the amount of context-sensitivity necessary for natural languages.

Mildly context-sensitive formalisms have the following properties:

1. They generate (at least) all CFLs.
2. They can describe a limited amount of cross-serial dependencies.

   In other words, there is a $n \geq 2$ up to which the formalism can generate all string languages $\{w^n \mid w \in T^*\}$.
3. They are polynomially parsable.
4. Their string languages are of constant growth.

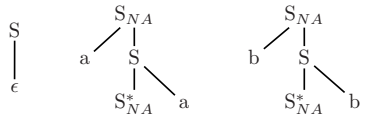
   In other words, the length of the words generated by the grammar grows in a linear way, e.g., $\{a^{2^n} \mid n \geq 0\}$ does not have that property.

Polynomial extensions of CFG (1)

Tree Adjoining Grammars (TAG), (Joshi, Levy, and Takahashi, 1975; Joshi and Schabes, 1997):

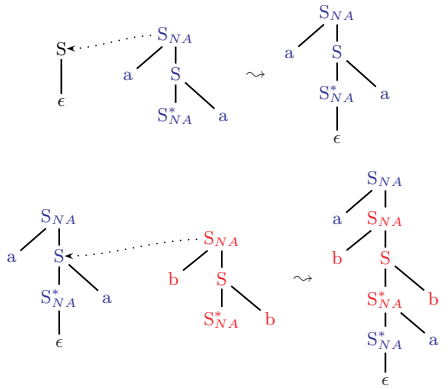
- Tree-rewriting grammar.
- Extension of CFG that allows to replace not only leaves but also internal nodes with new trees.
- Can generate the copy language.

Example: TAG for the copy language



Polynomial extensions of CFG (2)

Example: TAG derivation of *abab*:



Polynomial extensions of CFG (3)

Linear Context-free rewriting systems (LCFRS) and the equivalent Multiple Context-Free Grammars (MCFG), (Vijay-Shanker, Weir, and Joshi, 1987; Weir, 1988; Seki et al., 1991)

Idea: extension of CFG where non-terminals can span tuples of non-adjacent strings.

Example: $yield(A) = \langle a^n b^n, c^n d^n \rangle$, with $n \geq 1$.

The rewriting rules tell us how to compute the span of the lefthand side non-terminal from the spans of the righthand side non-terminals.

$$A(ab, cd) \rightarrow \varepsilon \quad A(aXb, cYd) \rightarrow A(X, Y) \quad S(XY) \rightarrow A(X, Y)$$

Generated string language: $\{a^n b^n c^n d^n \mid n \geq 1\}$.

LCFRS is more powerful than TAG but still mildly context-sensitive.

Polynomial extensions of CFG (4)

Range Concatenation Grammar (RCG) (Boullier, 2000)

- RCG contains clauses of the form $A(\ldots) \to A_1(\ldots)\ldots A_k(\ldots)$ where $A, A_1, \ldots, A_k$ are predicates. Their arguments are words over the terminal and nonterminal alphabets.
- Intuition: The predicates characterize properties of strings. A derivation starts with $S(w)$ where S is a start predicate. If this can be reduced to the empty word (i.e., property S is true for w), then w is in the language.

Example: RCG for $\{a^{2^n} \mid n \geq 0\}$.

$S(a) \to \varepsilon \qquad S(XY) \to E(X,Y)S(X)$

$E(a,a) \to \varepsilon \quad E(aX,aY) \to E(X,Y)$

Polynomial extensions of CFG (5)

RCGs are simple if

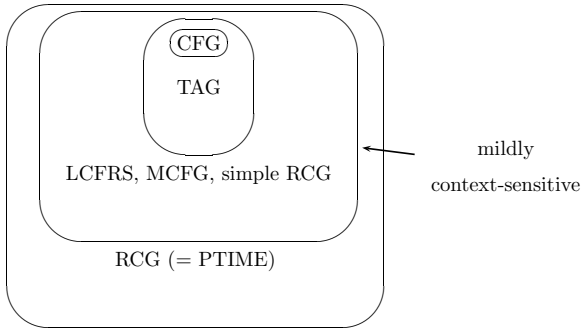
- the arguments in the right-hand sides of the clauses are single variables.
- no variable appears more than once in the left-hand side of a clause or more than once in the right-hand side of a clause.
- each variable occurring in the left-hand side of a clause occurs also in its right-hand side and vice versa.

Simple RCG are equivalent to LCFRS and MCFG.

RCG in general are more powerful; they generate exactly the class PTIME of polynomially parsable languages. (They properly include the class of MCS formalisms.)

Polynomial extensions of CFG (6)

Summary:



In this course, we are interested in mildly context-sensitive formalisms.

Basic Definitions: Languages (1)

Definition 1 (Alphabet, word, language)

1. *An alphabet is a nonempty finite set X.*
2. *A string $x_1 \ldots x_n$ with $n \geq 1$ and $x_i \in X$ for $1 \leq i \leq n$ is called a nonempty word on the alphabet X. X^+ is defined as the set of all nonempty words on X.*
3. *A new element $\varepsilon \notin X^+$ is added: $X^* := x^+ \cup \{\varepsilon\}$.*
   For each $w \in X^*$, the concatenation of w and ε is defined as follows: $w\varepsilon = \varepsilon w = w$.
   ε is called the empty word, and each $w \in X^*$ is called a word on X.
4. *A set L is called a language iff there is an alphabet X such that $L \subseteq X^*$.*

Basic Definitions: Languages (2)

Definition 2 (Homomorphism)

For two alphabets X and Y, a function $f : X^* \to Y^*$ is a homomorphism iff for all $v, w \in X^*$: $f(vw) = f(v)f(w)$.

Definition 3 (Length of a word) *Let X be an alphabet, $w \in X^*$.*

1. *The length of w, $|w|$ is defined as follows: if $w = \varepsilon$, then $|w| = 0$. If $w = xw'$ for some $x \in X$, then $|w| = 1 + |w'|$.*
2. *For every $a \in X$, we define $|w|_a$ as the number of as occurring in w: If $w = \varepsilon$, then $|w|_a = 0$, if $w = aw'$ then $|w|_a = |w'|_a + 1$ and if $w = bw'$ for some $b \in X \setminus \{a\}$, then $|w|_a = |w'|_a$.*

Basic Definitions: CFG (1)

Definition 4 (Context-free grammar)

A context-free grammar (CFG) is a tuple $G = \langle N, T, P, S\rangle$ such that

1. *N and T are disjoint alphabets, the nonterminals and terminals of G,*
2. *$P \subset N \times (N \cup T)^*$ is a finite set of productions (also called rewriting rules). A production $\langle A, \alpha\rangle$ is usually written $A \to \alpha$.*
3. *$S \in N$ is the start symbol.*

Basic Definitions: CFG (2)

Definition 5 (Language of a CFG)

Let $G = \langle N, T, P, S\rangle$ be a CFG. The (string) language $L(G)$ of G is the set $\{w \in T^* \mid S \overset{*}{\Rightarrow} w\}$ where

- *for $w, w' \in (N \cup T)^*$: $w \Rightarrow w'$ iff there is a $A \to \alpha \in P$ and there are $v, u \in (N \cup T)^*$ such that $w = vAu$ and $w' = v\alpha u$.*
- *$\overset{*}{\Rightarrow}$ is the reflexive transitive closure of $\Rightarrow$:*
  - *$w \overset{0}{\Rightarrow} w$ for all $w \in (N \cup T)^*$, and*
  - *for all $w, w' \in (N \cup T)^*$: $w \overset{n}{\Rightarrow} w'$ iff there is a v such that $w \Rightarrow v$ and $v \overset{n-1}{\Rightarrow} w'$.*
  - *for all $w, w' \in (N \cup T)^*$: $w \overset{*}{\Rightarrow} w'$ iff there is a $i \in \mathbb{N}$ such that $w \overset{i}{\Rightarrow} w'$.*

A language L is called context-free iff there is a CFG G such that $L = L(G)$.

Basic Definitions: CFG (3)

Proposition 1 (Pumping lemma for context-free languages)
Let L be a context-free language. Then there is a constant c such that for all $w \in L$ with $|w| \geq c$: $w = xv_1yv_2z$ with

- *$|v_1v_2| \geq 1$,*
- *$|v_1yv_2| \leq c$, and*
- *for all $i \geq 0$: $xv_1^iyv_2^iz \in L$.*

Basic Definitions: CFG (4)

Proposition 2 *Context-free languages are closed under homomorphisms, i.e., for alphabets T_1, T_2 and for every context-free language $L_1 \subset T_1^*$ and every homomorphism $h : T_1^* \to T_2^*$, $h(L_1) = \{h(w) \mid w \in L_1\}$ is a context-free language.*

Proposition 3 *Context-free languages are closed under intersection with regular languages, i.e., for every context-free language L and every regular language L_r, $L \cap L_r$ is a context-free language.*

Proposition 4 *The copy language $\{ww \mid w \in \{a, b\}^*\}$ is not context-free.*

Basic Definitions: Trees (1)

Definition 6 (Directed Graph)

1. *A directed graph is a pair $\langle V, E\rangle$ where V is a finite set of vertices and $E \subseteq V \times V$ is a set of edges.*
2. *For every $v \in V$, we define the in-degree of v as $|\{v' \in V \mid \langle v', v\rangle \in E\}|$ and the out-degree of v as $|\{v' \in V \mid \langle v, v'\rangle \in E\}|$.*

E^+ is the transitive closure of E and E^* is the reflexive transitive closure of E.

Basic Definitions: Trees (2)

Definition 7 (Tree)

A tree is a triple $\gamma = \langle V, E, r\rangle$ such that

- *$\langle V, E\rangle$ is a directed graph and $r \in V$ is a special node, the root node.*
- *γ contains no cycles, i.e., there is no $v \in V$ such that $\langle v, v\rangle \in E^+$,*
- *only the root $r \in V$ has in-degree 0,*
- *every vertex $v \in V$ is accessible from r, i.e., $\langle r, v\rangle \in E^*$, and*
- *all nodes $v \in V - \{r\}$ have in-degree 1.*

A vertex with out-degree 0 is called a leaf. The vertices in a tree are also called nodes.

Basic Definitions: Trees (3)

Definition 8 (Ordered Tree) *A tree is ordered if it has an additional linear precedence relation $\prec \in V \times V$ such that*

- *$\prec$ is irreflexive, antisymmetric and transitive,*
- *for all v_1, v_2 with $\{\langle v_1, v_2\rangle, \langle v_2, v_1\rangle\} \cap E^* = \emptyset$: either $v_1 \prec v_2$ or $v_2 \prec v_1$ and if there is either a $\langle v_3, v_1\rangle \in E$ with $v_3 \prec v_2$ or a $\langle v_4, v_2\rangle \in E$ with $v_1 \prec v_4$, then $v_1 \prec v_2$, and*
- *nothing else is in $\prec$.*

We use Gorn addresses for nodes in ordered trees: The root address is ε, and the jth child of a node with address p has address pj.

Basic Definitions: Trees (4)

Definition 9 (Labeling) *A labeling of a graph $\gamma = \langle V, E\rangle$ over a signature $\langle A_1, A_2\rangle$ is a pair of functions $l : V \to A_1$ and $g : E \to A_2$ with A_1, A_2 possibly distinct.*

Definition 10 (Syntactic tree) *Let N and T be disjoint alphabets of non-terminal and terminal symbols. A syntactic tree (over N and T) is an ordered finite labeled tree such that $l(v) \in N$ for each vertex v with out-degree at least 1 and $l(v) \in (N \cup T \cup \{\varepsilon\})$ for each leaf v.*

Basic Definitions: Trees (5)

Definition 11 (Tree Language of a CFG) *Let $G = \langle N, T, P, S\rangle$ be a CFG.*

1. *A syntactic tree $\langle V, E, r\rangle$ over N and T is a parse tree in G iff*
   - *$l(v) \in (T \cup \{\varepsilon\})$ for each leaf v,*
   - *for every $v_0, v_1, \ldots, v_n \in V$, $n \geq 1$ such that $\langle v_0, v_i\rangle \in E$ for $1 \leq i \leq n$ and $\langle v_i, v_{i+1}\rangle \in \prec$ for $1 \leq i < n$, $l(v_0) \to l(v_1) \ldots l(v_n) \in P$.*
2. *A parse tree $\langle V, E, r\rangle$ is a derivation tree in G iff $l(r) = S$.*
3. *The tree language of G is*

$$L_T(G) = \{\gamma \mid \gamma \text{ is a derivation tree in } G\}$$

Basic Definitions: Trees (6)

Definition 12 (Weak and Strong Equivalence)

Let F_1, F_2 be two grammar formalisms.

- *F_1 and F_2 are weakly equivalent iff for each instance G_1 of F_1 there is an instance G_2 of F_2 that generates the same string language and vice versa.*
- *F_1 and F_2 are strongly equivalent iff for both formalisms the notion of a tree language is defined and, furthermore, for each instance G_1 of F_1 there is an instance G_2 of F_2 that generates the same tree language and vice versa.*

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