

# An Analysis of Quantifier Scope Restrictions in Dependence Logic

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## Abstract

In our contribution we will present a strictly semantic approach to scope phenomena which is based on (Dynamic) Dependence Logic (DL) [Vää07, Gal13] and ideas from Dynamic Semantics [Fer93]. Formulas as well as quantifiers are interpreted as relations between sets of assignments. Scopal ambiguities are analyzed as a form of non-determinism: processing a formula in an input context  $X$  can lead to different output contexts  $Y$ . These outputs are constraint by imposing dependence relations on  $Y$ .<sup>1</sup>

## 1 Introduction: the data

One of the most serious challenges for formal theories of (quantified) NPs in natural language is the scopal behaviour of singular indefinites like ‘a’ or ‘some’ as well as numerical indefinites like ‘two’ or ‘three’ (see [WR11] for a recent survey). They not only give rise to wide-scope (WS) and intermediate scope (IS) readings (‘upward unboundedness’) but they can in addition escape both clausal and island boundaries, though their scope need not be maximal, resulting in the phenomenon of scopal ambiguity [Sza10, BF11].

- (1) a. Every man loves a woman.  
b. There is a woman who is loved by every man.
- (2) a. If some relative of mine dies I will inherit a house.  
b. [some relative of mine]<sub>*i*</sub> [if  $x_i$  dies, I will inherit a house]
- (3) a. A referee read three abstracts.  
b. There are three abstracts and each of them was read by a referee.

Example (4) shows that such indefinites can even have intermediate scope if the sentence contains three quantifier phrases.

- (4) a. Every student read a<sub>*x*</sub> paper that every<sub>*y*</sub> professor recommended.  
b. Every<sub>*x*</sub> student read every<sub>*y*</sub> paper that a<sub>*z*</sub> professor recommended.

Whereas in (4a) ‘a paper’ can semantically scope over ‘every student’, the universal ‘every professor’ in the relative clause cannot scope over the indefinite ‘a paper’. By contrast, for (4b) the indefinite ‘a professor’ in the relative clause can scope over both universals in the matrix clause, resulting in the following three readings where ‘>’ indicates scopal order: (i)  $x > y > z$  (narrow scope NS); (ii)  $x > z > y$  (intermediate scope IS) and (iii)  $z > x > y$  (wide scope WS). However, in simple sentences in English a universal NP in object position can scope over an indefinite in subject position, witness example (5) (for this sentence, the inverse scope reading is even more prominent than the direct scope reading [WR11]).

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(5) Some inhabitant of every city participated.

‘Upward unboundedness’ is not valid for all indefinites: modified numerals like ‘at least n’, ‘fewer than n’ or ‘exactly n’ do not allow such a WS reading, as shown by the following examples.

- (6) a. Two referees read more than three abstracts.  
b. \*There are more than three abstracts such that each of them was read by two (possibly different) referees.
- (7) a. At least three men read few books, possibly different ones.  
b. \*Few books are such that at least three (possibly different) men read them.

Second, a distinction between existential and distributive scope has to be made ([WR11], [Sza10, 92f.]).

(8) If three relatives of mine die, I will inherit a house.

(8) can only have the interpretations that (i) if the number of my dead relatives reaches three, I inherit a house (whichever relatives of mine passes away) or that (ii) there are three particular relatives such that if they all die, I will inherit a house. On both interpretations the total of houses inherited is one and not three, as expected on a distributive reading: there are three relatives of mine such that if any of them dies I will inherit a house (a total of three houses).

In the literature many of the above examples are analyzed using choice functions, i.e. 0-ary Skolem functions. For example, (2) can be analyzed as (9).

(9)  $\exists F_{<0>}[\text{if } [F_{<0>}(\lambda x x \text{ is a relative of mine})] \text{ dies, I will inherit a house}]$

Instead of using a variable over elements of the domain, a higher-order variable over choice functions with maximal scope is used [Sza10, 93], [Sch06, 300]. As shown in [Sza10, Sch06, Chi01], this mechanism is in general not expressive enough. [Chi01] observes that (10a) can be denied by using (10b) so that the quantifier over a choice function does not have widest scope.

- (10) a. Every linguist studied every solution that some problem might have.  
b. Not every linguist studied every solution that some problem might have.

A second problem is discussed in [Sza10, 95], showing that there is an inseparability of existential scope and the property denoted by the NP. If (11a) is analyzed as (11b), using material implication, a choice-functional analysis makes wrong predictions because the existence of a non-invited philosopher makes (11a) true.

- (11) a. If we invite a certain philosopher, Max will be offended.  
b.  $\exists F[\text{invite}(F(\text{philosopher}), \text{we}) \rightarrow \text{offended}(m)]$

## 2 Scope ambiguity as differences in dependence relations

In traditional approaches the ‘upward unboundedness’ of indefinites is explained in terms of linear quantifier scope relations. This is achieved either by using type-shifting rules, the effect of which consists in changing the linear order of quantifiers or by interpreting indefinites in situ by generalized Skolem functions in the lexicon and thereby not as ‘bona fide’ quantifiers [BF11].

According to [Sza97, 109], theories of scope which are based on mechanisms like quantifier raising, storage or type shifting are *semantically blind* because they use a single *syntactic* rule of scope assignments:  $\alpha_{[D...}\beta...]} \rightarrow \alpha \text{ scopes over } \beta$ . According to this rule, an expression  $\alpha$  is

‘prefixed’ to a domain  $D$  in order to assign scope to it over  $D$ , irrespective of what  $\alpha$  means and of what operator  $\beta$  may occur in  $D$ .

In contrast to those approaches, we will develop a strictly semantical approach which builds on the following assumptions: (i) formulas are interpreted as (possibly dynamic) relations between sets of assignments, (ii) following Dynamic Dependence Logic [Gal13] and [Fer93], quantifier expressions too are interpreted as relations between sets of assignments. On this perspective, two types of relational properties can be distinguished which are related to the cardinality information expressed by the quantifier and the notion of spawning a set of processes (reachability sets in the sense of [Fer93]). either conjunctively or disjunctively, (iii) different types of relational properties of a quantifier give rise to different forms of dependencies, which can be expressed in Dynamic Dependence Logic and (iv) a quantifier imposes these relational properties as *strategies* in the sense of Game Theory (Logic), which are formulated in terms of various dependence relations from Dynamic Dependence Logic and Database Theory.

Using such a dynamic formalism makes it possible to express dependence relations that hold either between variables (or attributes) of a single relation or between combinations of relations. Non-determinism can arise at the level of a single relation (inverse scope readings) or at the level of combinations of relations (IS readings).

In our semantic-driven approach verb stems are interpreted as sets of events. Each event denoted by a verb stem is related to a tuple of objects which can be singled out, e.g., in terms of thematic roles like actor or theme. On this perspective, the elements of a team are events and the values of the attributes are objects involved in the events so that each verb determines a team at the semantic level. Given a set of events of the same type (say readings), there can be various dependency relations between the roles defined by events of that type. For example, in case of a set of reading events, at least three different dependency relations can be distinguished. By way of illustration, assume the set of students to be  $\{s_1, s_2\}$  and the set of papers read to be  $\{p_1, p_2\}$ . Consider first the sentence ‘Every student read a paper’ with quantifier scope  $\forall\exists$ .

(12) Every student read a paper.

For  $\exists$ , there are two different strategies: either  $\exists$  is assigned a constant value or not. If the first strategy applies, this corresponds to a WS reading, which is shown in Table 1(a). In Dependence Logic, this is a special case of functional dependence, which can be expressed by  $=(t)$  (see definition (21b) below) if  $t = \textit{theme}$  (or ‘paper’ in the present case) because the values of the attribute THEME are constant with respect to the values of the attribute ACTOR.

(a) WS $F^{\text{paper}}(s_1) = \{p_1\}$ $F^{\text{paper}}(s_2) = \{p_1\}$	(b) NS1 $F^{\text{paper}}(s_1) = \{p_1, p_2\}$ $F^{\text{paper}}(s_2) = \{p_2\}$	(c) NS2 $F^{\text{paper}}(s_1) = \{p_1, p_2\}$ $F^{\text{paper}}(s_2) = \{p_1, p_2\}$
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Table 1: Different combinations for the scope sequence  $\forall\exists$

The other two tables are instances of non-(strict-)functional dependencies, corresponding to a NS-reading. These two examples differ with respect to a weak form of dependence, which can be expressed using the tensor operator.

(13)  $M \models_{(X,Y)} \phi \otimes \psi$  iff there are teams  $X_1, X_2$  s.t.  $X = X_1 \cup X_2, M \models_{(X_1,Y)} \phi$  and  $M \models_{(X_2,Y)} \psi$ .

In (13) the following additional constraint on the subteams  $X_1$  and  $X_2$  must be imposed: if the variables (attributes) of the team  $X$  are  $x_1, \dots, x_n$  (in that order), then the values for  $x_1$  must

be the same for  $X$ ,  $X_1$  and  $X_2$ . The idea behind using the tensor is that a team can be split into  $n$  subteams such that for each subteam a FD between the attributes ‘student’ and ‘paper’ holds. This form of dependence will be called *weak functional dependence* (wFD). Table 1(c) is an example of a wFD. This table also shows that the scope sequence  $\forall\forall$  is an instance of wFD. Consider next a sentence with surface order  $\exists\forall$ .

(14) A student read every paper.

For (14), the same argument applies as for (12). If for  $\exists$  a strategy is chosen which assigns to the corresponding argument a constant value, one gets the surface scope order  $\exists\forall$ , otherwise this yields the WS reading corresponding to  $\forall\exists$ .

Using the above examples, one arrives at the following preliminary thesis: *a WS reading is already possible if a quantifier  $Q$  admits both of a constant and a non-constant strategy (i.e. it shows (weak) variation)*. This thesis is too weak because it also holds for quantifiers like ‘at least’, which do not admit of a WS reading in object position.

(15) Every student read at least two papers.

Suppose there are four students  $\{s_1, s_2, s_3, s_4\}$  and three papers  $\{p_1, p_2, p_3\}$ . A possible distribution between students and papers is given in (16).

- (16) a.  $F^{paper}(s_1) = \{p_1, p_2\}$   
 b.  $F^{paper}(s_2) = \{p_1, p_3\}$   
 c.  $F^{paper}(s_3) = \{p_2, p_3\}$   
 d.  $F^{paper}(s_4) = \{p_1, p_2, p_3\}$

What is needed, therefore, is an additional requirement on the (non-)determinism of the cardinality information.

- (17) a.  $\forall : \underbrace{\alpha \otimes \dots \otimes \alpha}_{n\text{-times}}$  if  $card([[N]]) = n$   
 b. *at least*  $n : \underbrace{\alpha \otimes \dots \otimes \alpha}_{\geq n\text{-times}}$ . The number of subteams is not determined in order to fix a unique dependence relation.

The upshot of the above discussion is that two types of non-determinism must be distinguished: (i) non-determinism with respect to the cardinality, called *cardinality (non-)determinism* and (ii) non-determinism with respect to the object assigned to an attribute ( $\exists$  vs. *john*, random vs. non-random assignment), called *value (non-)determinism*. Cardinality (non-)determinism can be defined using wFD. In this case the cardinality of  $F^x(y_i)$  must be the same for each  $y_i$ . Value (non-)determinism can be defined as FD. In this case only outputs are admissible for which the value of the variable (attribute) of the quantifier, say  $x$ , is constant so that  $=(x)$  holds. Using these two types of non-determinism, one can define the notion of *weak variation* as follows.

- (18) A quantifier  $Q$  shows *weak variation* iff it satisfies (i) cardinality determinism and (ii) value determinism.

Using this definition of weak variation, one arrives at the following three theses: (i) *The surface order of quantifiers is always admissible*, (ii) *if the relational properties of a quantifier  $Q$  admit of weak variation, a (weak or strong) FD is an admissible strategy so that an inverse (WS) reading is possible* and (iii) *the set of dependence relations that can hold between two*

quantifiers is dependent on the relational properties of those quantifiers in terms of cardinality (non-)determinism and value (non-)determinism.

## 2.1 Intermediate readings and syntactic islands

The theory outlined so far is unable to account for the difference in scopal behaviour between indefinites and universal quantification.

- (19) a. Every<sub>x</sub> student read a<sub>y</sub> paper that every<sub>z</sub> professor recommended.  
 b. Every<sub>x</sub> student read every<sub>y</sub> paper that a<sub>z</sub> professor recommended.

The following two problems arise: (i) unlike the cases considered so far, there is no weak FD for the IS-reading of (19b). Second, for both (19a) and (19b) an MVD is possible (see below (21c) for the definition). In order to solve these problems one has to take into consideration that in both cases *two* relations are combined: the ‘read’ and the ‘recommend’ relation.

Beginning with example (19a) and assuming a set of two students  $\{s_1, s_2\}$ , a set of two papers  $\{p_1, p_2\}$  and three professors  $\{t_1, t_2, t_3\}$ , one gets for the function  $F^t(s_i, p_j) : F^t(s_i, p_j) = \{t_1, t_2, t_3\}$ . For (19a),  $F^t(s_i, p_j)$  is therefore always the whole set of professors so that no variation is possible. By contrast, for (19b) there are three different patterns, each corresponding to one of the three admissible scope readings, yielding the three patterns shown in Table 2.

(a) NS $F^t(s_1, p_1) = \{t_1\}$ $F^t(s_2, p_1) = \{t_2\}$ $F^t(s_1, p_2) = \{t_3\}$ $F^t(s_2, p_2) = \{t_1\}$	(b) WS $F^t(s_1, p_1) = \{t_1\}$ $F^t(s_1, p_2) = \{t_1\}$ $F^t(s_2, p_1) = \{t_1\}$ $F^t(s_2, p_2) = \{t_1\}$	(c) IS $F^t(s_1, p_1) = \{t_1\}$ $F^t(s_1, p_2) = \{t_1\}$ $F^t(s_2, p_1) = \{t_2\}$ $F^t(s_2, p_2) = \{t_2\}$
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Table 2: Different combinations for the surface scope sequence  $\forall\forall\exists$

The example in Table 2(a) corresponds to a NS reading. In this case there is no dependence between papers and professors. The second example (Table 2b) shows a WS reading for ‘professor’ since the value for the attribute is constant. The third example (Table 2c) corresponds to an IS reading. The  $t_i$  are distributed in such a way that they are constant with respect to the first (student) argument. In our theory this difference between  $\forall$  and quantifiers corresponding to indefinites can be explained as follows. First, the following thesis about the join of two relations is assumed, using the notion of *totality* defined below. The second condition in (20) excludes  $\forall$  because it always denotes the top element of the lattice of plural objects using a Link-style representation.

*Join of two relations:* In the composition of two relations  $R_1$  and  $R_2$  a quantifier  $Q$  occurring in  $R_2$  is *upward unbounded* only if (i) it admits of weak variation, (ii) there is a MVD between  $Q$  and a different quantifier occurring in  $R_1$  and (iii) this MVD is not *total*.

- (20) A quantifier  $Q$  is *total* iff (i)  $Q$  shows weak variation and (ii) the output  $Y$  is cardinality non-deterministic.

## 2.2 An outline of the formal theory

Dependence Logic, DL, [Vää07, Gal13] extends the syntax of FOL with dependence formulas which make it possible to express dependence relations between variables. E.g. the formula

$=(x, y)$  has the intuitive meaning that  $y$  is determined by  $x$ . Such an assertion only makes sense for sets of assignments and not for a single assignment. Therefore, in DL formulas are interpreted w.r.t. sets of assignments which are called *teams*.

[Gal13] observes that DL can be thought of as a logic of imperfect information and shows that it is possible to develop a dynamic variant, called *Dynamic Dependence Logic (DDL)*, in which expressions are interpreted as transitions between teams. This variant is based on the following two observations. First, in DL quantifiers are already implicitly interpreted as transitions  $X \rightarrow \Phi_M$  where  $\Phi_M$  is a local quantifier. In case this quantifier possibly leads to different outputs, one gets what is called a spawning of processes (transitions) or events running in parallel ([Fer93]). Second,  $\phi \sqcup \psi$  can be interpreted as the non-deterministic choice between doing either  $\phi$  or  $\psi$ . In DDL the satisfaction relation  $\models$  is defined as a relation between pairs of teams, a DL formula and a model. FOL literals and dependence atoms are interpreted as tests. The difference between FOL literals like  $R(x_1, \dots, x_n)$  and a dependence atom like  $=(x, y)$  is that the former are interpreted in the usual Tarskian sense. Multi-valued dependence  $\Rightarrow(x, y)$  is defined in (21c), whereas sequential composition ‘;’ is defined in (21d).

- (21) a.  $M \models_{(X,Y)} R(x_1, \dots, x_n)$  iff  $X = Y$  and for all  $s \in X : M \models_s R(x_1, \dots, x_n)$  where  $\models_s$  is the satisfaction relation in the usual (Tarskian) sense.  
b.  $M \models_{(X,Y)} =(x, y)$  iff  $X = Y$  and for all  $s, s' \in X$  if  $s(x) = s'(x)$  then  $s(y) = s'(y)$ .  
c.  $M \models_{(X,Y)} \Rightarrow(\bar{x}, \bar{y})$  iff  $X = Y$  and for all  $y \in \bar{y}$ ,  $F^y$  only depends on the values of  $\bar{x}$  where  $\bar{x}$  and  $\bar{y}$  are sequences of variables.  
d.  $M \models_{(X,Y)} \phi; \psi$  iff there exists a team  $Z$  s.t.  $M \models_{(X,Z)} \phi$  and  $M \models_{(Z,Y)} \psi$ .  
e.  $\phi \sqcup \psi := \exists x_1 \exists x_2 (=(x_1) \wedge =(x_2) \wedge ((x_1 = x_2 \wedge \phi) \otimes (x_1 \neq x_2 \wedge \psi)))$ .

Constancy of the value of a variable  $x$  is expressed by the dependence formula  $=(x)$ . The formula  $=()$  holds for all teams. Below the definitions for the quantifiers  $\exists, \forall$  and  $\exists^{\geq 5}$  are given (see [Eng12], [Gal13] for details).

- (22) a.  $M \models_X \exists x \phi$  iff there is a function  $F : X \rightarrow \exists_M$  such that  $M \models_{X[F/x]} \phi$ , where  $\exists_M$  is the local existential quantifier defined by  $\{A \subseteq M \mid A \neq \emptyset\}$  and  $X[F/x]$  is the team  $\{s[a/x] \mid s \in X, a \in F(s)\}$ .  
b.  $M \models_X \forall x \phi$  iff there is a function  $F : X \rightarrow \forall_M$  such that  $M \models_{X[F/x]} \phi$ , where  $\forall_M$  is the local universal quantifier defined by  $\{A \subseteq M \mid A = M\}$ .  
c.  $M \models_X \exists^{\geq 5} x \phi$  iff there is a function  $F : X \rightarrow \exists_M^{\geq 5}$  such that  $M \models_{X[F/x]} \phi$ , where  $\exists_M^{\geq 5}$  is the local cardinality quantifier defined by  $\{A \subseteq M \mid |A| \geq 5\}$ .

According to (22a), non-determinism is built into the definition of the existential quantifier because it allows the choice of an arbitrary non-empty set of witnesses for an existentially quantified variable. This is called *lax semantics* in DL. The definitions in (22) are static because a quantifier is interpreted with respect to a team and not a pair of teams. In (23) the definition of the dynamic variant of  $\exists$  is given.

- (23)  $M \models_{(X,Y)} \exists x$  iff there is a function  $F \rightarrow \exists_M$  s.t.  $X[F/x] \subseteq Y$ .

The formula  $\exists x \phi$  from DL is translated into DDL by  $\exists x; \phi'$  with  $\phi'$  the translation of  $\phi$ .  $\exists x; \phi'$  requires the whole output team  $Y$  to be input to  $\phi'$ . On this interpretation of a quantifier, the differences with respect to the relational properties expressible in terms of cardinality- and value (non-) determinism are not taken into account. Such constraints can be imposed by using the dependence formulas corresponding to the particular type of (non-) determinism. In (24a),

the general case is given, whereas (24b) holds for  $\exists$ .

$$(24) \quad \begin{array}{l} \text{a. } \Phi x; (\psi_1 \sqcup \dots \sqcup \psi_n); \phi'. \\ \text{b. } \exists x; (= (x) \sqcup = ()); \phi'. \end{array}$$

In (24a),  $\Phi$  is a quantifier and  $\psi_i$ ,  $1 \leq i \leq n$ , is a dependence formula. According to (24b), in the case of  $\exists$  (non-deterministically) either the value of the newly introduced element is chosen to be constant,  $=(x)$ , or no constraint is imposed,  $=()$ . On this approach,  $(\psi_1 \sqcup \dots \sqcup \psi_n)$  is part of the interpretation of a quantifier, reflecting the possible choices of dependence relations it admits. Therefore, a quantifier (possibly) triggers a non-deterministic choice of how the values of variables introduced by it are determined in the interpretation of an expression in which it occurs. Different choices correspond to what is analyzed as a difference in the linear order of quantifiers in other approaches.

Sentences like (4a) and (4b) are analyzed as the  $\cap$ -combination of two relations  $R_1$  (say ‘read’) and  $R_2$  (say ‘recommend’). For the sake of simplicity,  $\cap$  is defined only for two 2-place relations with  $\phi = R_1(x, y)$  and  $\psi = R_2(z, y)$ .

$$(25) \quad M \models_{(X,Y)} \phi \cap \psi \text{ iff there are } Y_1 \text{ and } Y_2 \text{ s.t. (i) } M \models_{(X,Y_1)} \phi, \text{ (ii) } M \models_{(X,Y_2)} \psi \text{ and (iii) } (Y = \{\langle a_x, b_y, c_z \rangle \mid \langle a_x, b_y \rangle \in Y_1 \text{ and } \langle c_z, b_y \rangle \in Y_2\}) \text{ and there is a FD between the two relating variables, here } x \text{ and } z).$$

According to (25),  $Y$  is the join of two binary relations. Since both relations are (possibly) non-deterministic, the output team  $Y$  is not uniquely determined. The output team  $Y$  can be classified according to the FD’s and MVD’s that are possible according to the surface order of quantifiers. Consider the surface order  $\forall\forall\exists$  ( $=$  (19b)). For  $R_1$  ( $=$  ‘read’), one has (26).

$$(26) \quad \{(M, R_1) \mid \exists A \in Q_1, \exists B \in Q_2, A \times B \subseteq R_1\}.$$

Thus, the output team for  $R_1$  simply is  $M_{|student} \times M_{|paper}$ , where  $M_{|student}$  and  $M_{|paper}$  are the restrictions of the domain to students and papers, respectively. As a consequence,  $R_1$  is a total relation between  $M_{|student}$  and  $M_{|paper}$ . For  $R_2$ , one has the three principal possibilities depicted in Table 3.

(a) WS $F^{\text{professor}}(p_1) = \{t_1\}$ $F^{\text{professor}}(p_2) = \{t_1\}$	(b) NS1 $F^{\text{professor}}(p_1) = \{t_1, t_2\}$ $F^{\text{professor}}(p_2) = \{t_1, t_3\}$	(c) NS2 $F^{\text{professor}}(p_1) = \{t_1, t_2\}$ $F^{\text{professor}}(p_2) = \{t_1, t_2\}$
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Table 3: Different combinations for *A professor recommended a paper*

If the output team of  $R_2$  satisfies the FD  $=(\textit{professor})$ , Table 3(a), i.e. the value of the variable *professor* is constant, one gets a WS reading for the corresponding quantified NP ‘a professor’. If  $R_2$  does not impose the constraint  $=(\textit{professor})$ , two principle cases have to be distinguished, which are depicted in Table 3(b) and Table 3(c). If the output team is like that in Table 3(b) (NS1), one gets a NS reading, corresponding to Table 2(a). By contrast, if the output team is like that in Table 3(c), one gets the IS reading depicted in Table 2(c) if clause (iii) in (25) is used with  $=(x, z)$  s.t.  $F(s_i) = t_i$ , with  $F$  a function expressing the relation between students and professors. Thus, for the join of two relations, the different scope possibilities simply follow from the strategies imposed by the quantifiers occurring in the relations  $R_1$  and  $R_2$  and the way these relations are combined by  $\cap$ .

In our theory the relational properties of quantifiers depending on both cardinality- and value (non-) determinism give rise to different functional dependence relations which can be used by

a speaker to interpret a sentence, resulting in different scope sequences according to traditional theories. It is important to note that such a strategy is only possible if quantifiers are interpreted as relations between sets of assignments because only then is it possible to impose dependence relations on the output team. In this respect, our approach is similar to that of [BF11]. In contrast to approaches based on (generalized) Skolem functions, there is no separation between the existential scope and the property denoted by the NP so that the problem related to (11) does not arise. The fact that existential and distributional scope are different is explained as follows. A sentence like (8) is analyzed as the join of two relations. For bare numerical indefinites like ‘three’, both a distributive and a collective reading are possible. The thesis, now, is that (8) has to be analyzed in a way similar to the examples in (19). Recall that in the former case an IS reading is possible only if the quantifier triggering this reading shows strong variation. This type of variation is guaranteed if the quantified NP (‘three relatives’) gets a collective reading (only one plural object). By contrast, on a distributive interpretation, strong variation fails if one assumes that bare numerical indefinites are total, similarly to  $\forall$ . One way of establishing this thesis consists in arguing that on a distributive reading for a bare numerical indefinite like ‘three relatives’ the function  $F^{relative}$  always yields the whole set of three relatives as output, similarly to  $\forall$  (see full paper for details).

It goes without saying that a lot of work remains to be done. First, the formal details must be worked out in a rigorous way (see full paper for details). Second, there are at least the following empirical questions which have to be answered: (i) How is the fact explained that ‘exactly’ does not show upward unboundedness?, (ii) How can such principles as the Binder Roof Constraint and the No Skipping Constraint be incorporated into the theory (see [BF11])? and (iii) The definition of the join operator  $\cap$  does not take into account semantic differences between, say, relative clauses and conditionals. How can  $\cap$  be used in a compositional way? This requires, at least to incorporate a component to handle both intra- and intersentential anaphora into the theory. We are currently working on integrating the current theory with the sequential semantics of Vermeulen and Van Eijck’s Incremental Dynamics.

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